Delhi School of Economics
Department of Economics

Entrance Examination for M. A. Economics
Option B
June 28, 2008

Time 3 hours Maximum marks 100

General instructions. Please read the following instructions carefully.

- Check that your examination has pages 1 to 5 and you have been given a blank Answer booklet. Do **not** start writing until instructed to do so by the invigilator.
- Fill in your Name and Roll Number on the small slip attached to the Answer booklet. Do **not** write this information anywhere else in this booklet.
- When you finish, hand in this Examination along with the Answer booklet to the invigilator.
- Do not disturb your neighbors at any time. **Anyone engaging in illegal examination practices will be immediately evicted and that person's candidature will be cancelled.**
- The examination has two sections. Follow the instructions given at the beginning of each section.

Do not write below this line.

This space is for official use only.

Fictitious Roll Number

Section/Question	Marks
I	
II/11	
II/12	
II/13	
II/14	
II/15	
Total	

197

Section I

Instructions. *Answer Questions 1 to 10. Each question is followed by four assertions, (a) to (d), one of which is correct. Indicate the correct choice by circling it on this Examination. Each correct choice will earn you 2 marks. However, you will lose 2/3 mark for each incorrect choice.* **Please ensure that you hand in this Examination along with the Answer booklet.**

Question 1. There is a pile of 17 matchsticks on a table. Players 1 and 2 take turns in removing matchsticks from the pile, starting with Player 1. On each turn, a player has to remove a number of sticks that equals the square of a positive integer, such that the number of matchsticks that remain on the table equals some non-negative integer. The player who cannot do so, when it is his turn, loses.

(a) If Player 2 plays appropriately, he can win regardless of how 1 actually plays.
(b) If Player 1 plays appropriately, he can win regardless of how 2 actually plays.
(c) Both players have a chance to win, if they play correctly.
(d) The outcome of the game cannot be predicted on the basis of the data given.

Question 2. Given $A \subset \Re$, let 1_A be the function defined on $\Re$ by: $1_A(x) = 1$ if $x \in A$ and $1_A(x) = 0$ if $x \notin A$. Consider the sequence of functions (f_n) for positive integers n, where $f_n(x) = n1_{[0,1/n]}(x)$ for $x \in \Re$.

(a) For every x, the sequence of numbers $(f_n(x))$ has a limit in $\Re$.
(b) For every x, $\lim_{n\to\infty} f_n(x)$ does not exist.
(c) If $\lim_{n\to\infty} f_n(x)$ exists, then the limit depends on x.
(d) $\lim_{n\to\infty} f_n(x)$ exists for all but a finite set of $x \in \Re$.

Question 3. There are three identical boxes, each with two drawers. Box A contains a gold coin in each drawer. Box B contains a silver coin in each drawer. Box C contains a gold coin in one drawer and a silver coin in another drawer. A box is chosen, a drawer opened and a gold coin is found. What is the probability that the chosen box is C?

(a) 2/3
(b) 1/3
(c) 1/2
(d) 3/4

Question 4. A random variable has outcomes Success and Failure with probabilities 3/4 and 1/4 respectively. A gambler observes the sequence of outcomes of this variable and receives a prize of 2^n if n is the first time that Success occurs. What is the expected value of the gambler's prize?

(a) 1

(b) 2

(c) 3

(d) 4

Question 5. A number of mathematicians in the middle of the 20th century contributed to a series of books published in the name of a fictitious mathematician called Bourbaki. Suppose a sociological critic of science asserts "Every book by Bourbaki contains a chapter such that the validity of every theorem in that chapter depends on the reader's gender." If every assertion by this critic is false, which of the following assertions must be true?

(a) There exists a book by Bourbaki containing a chapter such that the validity of every theorem in that chapter is independent of the reader's gender.

(b) Every chapter in every book by Bourbaki contains a theorem whose validity is independent of the reader's gender.

(c) There exists a book by Bourbaki such that every chapter in it contains a theorem whose validity is independent of the reader's gender.

(d) Every book by Bourbaki contains a chapter such that the validity of all the theorems in it is independent of the reader's gender.

Question 6. A family has 3 children. Suppose the probability that a child will be a girl is 1/2 and that all births are independent. If the family has at least one girl, what is the probability that the family has at least one boy?

(a) 5/7

(b) 6/7

(c) 5/6

(d) 4/6

Question 7. Suppose $f : \Re^2 \to \Re$ is continuously differentiable and $f(a, b) = 0$. A condition that ensures the existence of a unique continuously differentiable function ϕ such that $b = \phi(a)$ and $f(x, \phi(x)) = 0$ for all x sufficiently close to a is

(a) $D_1 f(a, b) > 1$

(b) $D_1 f(a, b) < 1$

(c) $D_2 f(a, b) > 1$

(d) $D_2 f(a, b) < 1$

Question 8. Suppose X and Y are independent random variables with standard Normal distributions. The probability of $X > 1$ is $p \in (0, 1)$ and the probability of $Y < -2$ is $q \in (0, 1)$. What is the probability of the event: $X^3 > 2$ **and** $Y^2 > 1$?

(a) pq

(b) p^2q

(c) pq^2

(d) $2pq$

Question 9. Suppose each of n players tosses a coin that has probability 1/2 of falling Heads and probability 1/2 of falling Tails. A player wins if her coin toss is Heads and the other players' coin tosses are Tails. This game is played until someone wins. The probability of finding a winner in the k-th iteration of the game is

(a) $2^{1-n}(1 - 2^{1-n})^{k-1}$

(b) $n2^{1-n}(1 - n2^{1-n})^{k-1}$

(c) $n2^{-n}(1 - n2^{-n})^{k-1}$

(d) $2^{-n}(1 - 2^{-n})^{k-1}$

Question 10. Consider the game in Question 9. If $n = 4$, then the probability of finding a winner in up to two iterations is

(a) 7/16

(b) 15/16

(c) 39/64

(d) 15/64

Section II

Instructions. *Answer any four of the following five questions in the Answer booklet.*

Question 11. (A) Prove that $d : \Re^n \times \Re^n \to \Re$, defined by $d(x, y) = \langle x - y, x - y \rangle^{1/2}$, is a distance function, where $\langle ., . \rangle$ denotes an inner product on $\Re^n$.

(B) Prove that, if (X, d) is a compact metric space and $f : X \to X$ is such that $d(x, y) = d(f(x), f(y))$ for all $x, y \in X$, then f is surjective.

(8, 12)

Question 12. (A) Prove that if (X, d) is a metric space, then the distance function d is continuous.

(B) Suppose (X,d) is a nonempty and complete metric space and $f : X \to X$ is a contraction, i.e., there exists $\beta \in (0,1)$ such that $d(f(x), f(y)) \leq \beta d(x,y)$ for every $x, y \in X$. Show that there exists exactly one $x \in X$ such that $f(x) = x$. (Hint: Start with some point in X and use f to construct a Cauchy sequence in X. Now use the completeness and contraction properties.)

(8, 12)

Question 13. Let $r_1 < r_2$, $\tau \in [r_1, r_2]$, and for every $t \in [r_1, r_2]$, let $A(t)$ be an $n \times n$ real matrix and let $b(t) \in \Re^n$. Suppose, for every $\psi \in \Re^n$, there exists one and only one differentiable function $\phi : [r_1, r_2] \to \Re^n$ such that $D\phi(t) = A(t)\phi(t)$ for every $t \in [r_1, r_2]$ and $\phi(\tau) = \psi$.

(A) Show that the space of solutions of the ordinary differential equation $Dx(t) = A(t)x(t)$ is an n-dimensional vector space.

(B) A collection $\{\phi^1, \ldots, \phi^m\}$ of solutions of $Dx(t) = A(t)x(t)$ is linearly independent if and only if $\{\phi^1(t), \ldots, \phi^m(t)\} \subset \Re^n$ is linearly independent for every $t \in [r_1, r_2]$.

(12, 8)

Question 14. $X \subset \Re^n$ is said to be a convex set if $x, y \in X$ and $t \in (0,1)$ imply $tx + (1-t)y \in X$. Given a convex set $X \subset \Re^n$, a function $f : X \to \Re$ is said to be convex if $x, y \in X$ and $t \in (0,1)$ imply $f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$.

Suppose $X \subset \Re^n$ is convex and $f : X \to \Re$ is continuous, where $\Re^n$ and $\Re$ are given the Euclidean metrics. Show that, if $f(x/2 + y/2) \leq f(x)/2 + f(y)/2$ for all $x, y \in X$, then f is a convex function.

(Hint: Use the fact that every $t \in (0,1)$ can be represented in the form $t = \sum_{r=1}^{\infty} \alpha_r 2^{-r}$ where each α_r is 0 or 1.)

(20)

Question 15. (A) Suppose an observation X is drawn from an unknown distribution P and that the following simple hypotheses are to be tested:

$$H_0 : \ P \text{ is a uniform distribution on the interval } (0,1)$$

$$H_1 : \ P \text{ is a standard normal distribution}$$

Determine the most powerful test of size 0.01 and calculate the power of the test when H_1 is true.

(B) Suppose $X_1, \ldots, X_n$ are a random sample from the uniform distribution on $[0, \theta]$, where the parameter θ is unknown. Derive the maximum likelihood estimator of θ.

(10, 10)

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