DSE-Type Exam

Your grade on this exam does not matter for anything except the decision about whether you need to read (or re-read) the topic in which you did not score well, so do not panic. You have 180 minutes to complete the test.

Part I (20 Questions – One point each)

1. *A function $y = f(x)$ is said to be an even function if $f(-x) = f(x)\ \forall x$. Consider the following functions: i) $f(x) = x$ and ii) $g(x) = |x|$*
   a) *Both f and g are even functions*
   b) *Only f is an even function*
   c) *Only g is an even function*
   d) *None of the two functions is even.*

2. *Let S be the set of integers. For $a, b \in S$, $(a, b) \in R$ if and only if $|a - b| < 1$. Consider the following statements:*
   i) $(a, a) \in R\ \forall a \in S$
   ii) If $(a, b) \in R$ then $(b, a) \in R$
   iii) $R = S \times S$

   Which of the following is correct?

   a) *Only i and ii are true*
   b) *Only i and iii are true*
   c) *i, ii and iii are true*
   d) *None of the above is true*

3. *If $P(A) > 0$, then the event A is independent of itself if and only if $P(A)$ is*
   a) *1/3*
   b) *1/2*
   c) *1*
   d) *None of these*

4. *"Capitalism is the astounding belief that the most wickedest of men will do the most wickedest of things for the greatest good of everyone" John Maynard Keynes*
   The idea in the above statement is supported by the following result in economic theory:
   a) *Fundamental Welfare Theorems*
   b) *Arrow's Impossibility Theorem*
   c) *Theory of Comparative Advantage*
   d) *Revealed preference theory*

5. *Suppose random variable X takes the value -1 with probability 3/10, 0 with probability 3/10, 1 with probability 3/10 and 2 with probability 1/10.*
   Define a random variable $Y = |X| = \max\{X, -X\}$ and $Z = \min\{X, -X\}$
   Which of the following is incorrect?

a) *Median (X) = 0, Median (|X|) = 1*
b) *Mean (X) = 1/5, Mean (|X|) = 4/5*
c) *E(|Y+Z|) > 0*
d) *Y = |Z|*

6. *Isoquants are downward sloping because*
   a) *Marginal rate of technical substitution is diminishing*
   b) *Both inputs have positive marginal products*
   c) *Prices of inputs are positive*
   d) *All of the above*

7. *Slope of the Engle curve for commodity X is negative. This implies that*
   a) *X is a giffen good*
   b) *X is a normal good*
   c) *X is bad*
   d) *X is an inferior good*

8. *The function $f(x) = |x|^2$ is*
   a) *continuous everywhere but not differentiable at $x = 0$*
   b) *not continuous at $x = 0$*
   c) *differentiable everywhere*
   d) *None of the above*

9. *Consider a function $f : R \rightarrow R$, where R denotes the set of real numbers. If f is decreasing, i.e., $x \geq y$ implies $f(x) \leq f(y)$, then*
   a) *f is concave*
   b) *f is convex*
   c) *f is quasi convex*
   d) *f is continuous*

10. *J curve is the term used for*
    a) *The typical time pattern of the response of net exports to a depreciation of the real exchange rate.*
    b) *The typical time pattern of the response of net exports to an appreciation of the real exchange rate.*
    c) *The typical time pattern of the response of net exports to an external supply shock.*
    d) *The typical time pattern of the response of net exports to an internal supply shock.*

11. *The alternative hypothesis is that a process is producing more than the maximum allowable proportion of defective pieces. A Type I error is to conclude that the process is*
    a) *Producing too many defective pieces when it actually isn't.*
    b) *Not producing too many defective pieces when it actually is not.*
    c) *Producing too many pieces when it actually is.*
    d) *Not producing too many too many defective pieces when it actually is.*

12. *The firms in a competitive industry operate with the production function $Y = (K + L)^{1/2}$, where Y is quantity of output, and K and L are quantities of capital and Labor respectively. Suppose that $w > r$. Expansion path is given by*
    a) $K = 0$
    b) $L = 0$
    c) $K = L$
    d) *None of the above*

13. *Consider the following function:*
    $f(x) = x^2 + 4x - 3$ *for* $x \in [1, 2]$
    $= 8x - 7$ *for* $x \in [2, 4]$
    Which of the following statements is true?
    a) $f(x)$ *is not defined at* $x = 2$
    b) $f(x)$ *is not continuous at* $x = 2$
    c) $f(x)$ *is not differentiable at* $x = 2$
    d) *None of the above*

14. *What is the greatest value assumed by the function* $y = 2 - |x - 2|$*?*
    a) 1
    b) 2
    c) 3
    d) ∞

15. *If the probabilities that A and B will die within a year are p and q, respectively. What is the probability that only one of them will be alive at the end of the year?*
    a) $p + q - pq$
    b) $p + q$
    c) $p + q - 2pq$
    d) *None of the above*

16. *Ten cards numbered from 1 to 10 are mixed in a hat. Two cards numbered p and q are drawn from the hat. What is the probability that* $p+q=10$*?*
    a) 2/5
    b) 4/9
    c) 7/45
    d) 4/45

17. *For the function defined by* $y = |x|$*, write an expression for dy/dx for each value of x where the derivative exists.*
    a) 1
    b) -1
    c) $-x/|x|$
    d) $x/|x|$

18. *In how many ways may N boys be lined up (B_1, B_2,….B_N) if B_1 must always come somewhere before B_2?*
    a) *N!*
    b) *(N-1)!*
    c) *(N-2)!*
    d) *N!/2*

19. *Function f(x) = max { x, 1 } is*
    a) *Not quasi convex*
    b) *Quasi convex but not convex*
    c) *Convex*
    d) *Concave*

20. *$\cap_n$ (-1/n, 1/n) is*
    a) *An interval*
    b) *A set consisting of two points*
    c) *Singleton*
    d) *None of the above*

Part II (40 Questions – 2 points each)

(Problems 1 to 5) In a perfectly competitive economy there are two goods, X and Y, produced using capital K and Labor L, according to the following production function:

$X = \min\{K_x, L_x\}$ *and* $Y = \min\{2K_y, 2L_y\}$

where K_x, L_x *are the inputs of K and L into the production of X and* K_y, L_y *are the inputs of K and L into the production of Y. There is a fixed supply of capital (200 units) and of labor (220 units), fully mobile between sectors.*

1. *The production possibility frontier of the above economy is given by*
   a) $X + 2Y = 200$
   b) $X + Y = 200$
   c) $X + Y/2 = 200$
   d) *None of the above*

Now suppose, there are two identical consumers (A and B) having preferences

$u_i = x_i^{1/2} y_i^{1/2}$ $i \in \{A, B\}$

2. *Given a production plan (x, y) on the production possibility frontier, consumption contract curve is given by*
   a) $\{((x_A, y_A),(x_B, y_B))|\ y_A = x_A,\ x_A + x_B = x \text{ and } y_A + y_B = y\}$
   b) $\{((x_A, y_A),(x_B, y_B))|\ y_A = (y/x)x_A,\ x_A + x_B = x \text{ and } y_A + y_B = y\}$
   c) $\{((x_A, y_A),(x_B, y_B))|\ y_A = (x/y)x_A,\ x_A + x_B = x \text{ and } y_A + y_B = y\}$
   d) $\{((x_A, y_A),(x_B, y_B))|\ y_A = 2x_A,\ x_A + x_B = x \text{ and } y_A + y_B = y\}$

3. *Given a production plan (x, y) on the production possibility frontier, utility possibility frontier is given by*
   a) $u_A + u_B = x^{1/2} y^{1/2}$
   b) $u_A + u_B = x + y$
   c) $u_A + u_B = xy$
   d) *None of the above*

4. *The grand utility possibility frontier is obtained by combining utilities* (u_A, u_B) *of the set of pareto efficient allocations (that is, where Marginal rate of Substitution of X and Y for the two individuals equal Marginal rate of Transformation of X and Y for the economy with efficiency in production)*
   a) $u_A + u_B = 300$
   b) $u_A + u_B = 200\sqrt{2}$
   c) $u_A + u_B = 100\sqrt{2}$
   d) *None of the above*

5. *Pareto efficient production plan(s) for this economy (i.e. a plan for which there exist an allocation where MRS =MRT) is(are)*
   a) *All points on the production possibility frontier*
   b) *Exactly one point on the production possibility frontier*
   c) *Exactly two points on the production possibility frontier*
   d) *None of the above*

6. *A consumer has a utility function* $u(x, y) = \max\{2x + y, x + 3y\}$. *He consumes (1, 0) in equilibrium. Then which of the following must be true?*
   a) $p_x \leq p_y$
   b) $p_x \leq (3/2)p_y$
   c) $p_x \leq (2/3)p_y$
   d) $p_x < p_y$

7. *There are two individuals A and B. The utility function of both individuals are identical and given by* $u(x, y) = \max\{x, y\} + (\max\{x, y\})^2$. *Each of them has 1 unit of x and 1 unit of y. Which of the following is a pareto efficient allocation?*
   a) *A has x = 1, y = 1 and B has x = 1 and y = 1*
   b) *A has x = 2, y = 0 and B has x = 0 and y = 2*
   c) *A has x = 0, y = 0 and B has x = 2 and y = 2*
   d) *None of the above*

(Problems 8 to 10) A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and when he flips it, it shows heads.

8. *What is the probability that it is a fair coin?*
   a) *0*
   b) *1/2*
   c) *1/3*
   d) *None of the above*

Suppose that he flips the same coin a second time and again it shows heads.

9. *Now what is the probability that it is a fair coin?*
   a) *0*
   b) *1/4*
   c) *1/5*
   d) *None of the above*

Suppose that he flips the same coin a third time and it shows tails.

10. *Now what is the probability that it is a fair coin?*

a) 0
b) 1/2
c) 1/3
d) 1

11. Suppose five fair coins are tossed. Let E be the event that all coins land heads. Define the random variable I_E

$I_E = 1$, if E occurs

$= 0$, if E^c occurs

Expected value of the random variable I_E is given by

a) 1
b) 1/2
c) 1/8
d) 1/32

12. Suppose that p(x, y), the joint probability mass function of X and Y, is given by

$y\downarrow \quad x\rightarrow$	1	2
1	0.5	0.1
2	0.2	0.2

The conditional probability mass function of X given that Y = 1 is

a) $p_{X|Y=1}(1) = 5/6$, $p_{X|Y=1}(2) = 1/6$
b) $p_{X|Y=1}(1) = 5/7$, $p_{X|Y=1}(2) = 2/7$
c) $p_{X|Y=1}(1) = 0.5$, $p_{X|Y=1}(2) = 0.1$
d) $p_{X|Y=1}(1) = 0.5$, $p_{X|Y=1}(2) = 0.2$

(Problems 13 to 14) Let X be uniformly distributed over (0, 1). Probability distribution function of X is given by $F(x) = P(X \leq x) = 0$, for $x \leq 0$

$= x$, for $x \in (0, 1)$
$= 1$, for $x \geq 1$

Define a new random variable $Y = X^2$. Let G be the probability distribution function of Y.

13. Probability distribution function of Y satisfies

a) $G(y) = F(\sqrt{y})$, for $y \in (0, 1)$
b) $G(y) = F(y^2)$, for $y \in (0, 1)$
c) $G(y) = (F(y))^2$, for $y \in (0, 1)$
d) $G(y) = F(y)$, for $y \in (0, 1)$

14. *Which of the following is true?*
    a) $G(x) < F(x) \ \forall x \in (0, 1)$
    b) $G(x) > F(x) \ \forall x \in (0, 1)$
    c) $G(x) = F(x) \ \forall x \in (0, 1)$
    d) *None of the above is true as sometimes G(x) exceeds F(x) and rest of the times F(x) exceeds G(x) for* $x \in (0, 1)$

15. *Expected sum obtained when three fair dice are rolled is*
    a) *12*
    b) *10.5*
    c) *9*
    d) *7.5*

16. *There is a pile of 18 matchsticks on a table. Players 1 and 2 take turns in removing matchsticks from the pile, starting with player 1. On each turn, a player has to remove a number of sticks that equals either 1, 2 or 3, such that the number of matchsticks that remain on the table equals some non-negative integer. The player, who cannot do so, when it is his /her turn, loses. Which of the following statements is true?*
    a) *If player 2 plays appropriately, he/she can win regardless of how 1 actually plays.*
    b) *If player 1 plays appropriately, he/she can win regardless of how 2 actually plays.*
    c) *Both players have a chance to win, if they play correctly.*
    d) *The outcome of the game cannot be predicted on the basis of the data given.*

(Problems 17 to 18) Suppose there are two profit maximizing firms who produce a homogeneous good at zero cost. The inverse demand function for this good is $p = 30 - q$, *where p is the price and q is the total output produced by these firms. Suppose the firms choose their outputs sequentially, with firm 1 choosing* q_1 *first, firm 2 choosing* q_2 *next (having seen* q_1*). If these firms behave rationally (i.e., act so as to maximize own profit) given their information and anticipate rational behavior by their rivals, then what will be the outputs chosen by the firms?*

17. *In equilibrium, outputs chosen by the firms are*
    a) $(q_1, q_2) = (15, 7.5)$
    b) $(q_1, q_2) = (25, 12.5)$
    c) $(q_1, q_2) = (10, 10)$
    d) *None of the above*

18. *Ratio of the profits of two firms is given by*
    a) $\pi_1/\pi_2 = 1$
    b) $\pi_1/\pi_2 = 1/3$
    c) $\pi_1/\pi_2 = 1/2$
    d) *None of the above*

(Problems 19 to 23) Consider the production function: $Q = \max \{K, L\}$

19. *Which of the following is true about Q?*

a) *Q is both quasi-convex and quasi-concave*
b) *Q is quasi-convex and not quasi-concave*
c) *Q is quasi-concave and not quasi-convex*
d) *Q is neither quasi-convex nor quasi-concave*

Define Cost function, C(w, r, Q) as minimum cost required to produce the output Q given input prices w per unit of labor and r per unit of capital and Input Demand functions L(w, r, Q) and K(w, r, Q) as solution to the optimization problem: Min wL + rK subject to Q = max {K, L}

20. *For Q above, the input demand function for L is given by*
    a) $L = QI_{\{w \geq r\}}$ *where* $I_{\{w \geq r\}}$ *takes the value 1 when* $w \geq r$ *and 0 otherwise*
    b) $L = QI_{\{w \leq r\}}$ *where* $I_{\{w \leq r\}}$ *takes the value 1 when* $w \leq r$ *and 0 otherwise*
    c) $L = Q$
    d) $L = 0$

21. *Cost function for the above problem is given by*
    a) $C = wL + rK$
    b) $C = (wr)^{1/2}Q$
    c) $C = \min\{w, r\}\, Q$
    d) $C = \max\{w, r\}\, Q$

22. *Given above production function, suppose p = 20, w = 25 and r = 18 the profit maximizing monopolist will produce*
    a) *0 units*
    b) *2 units*
    c) *10 units*
    d) *None of the above*

23. *Given above production function, suppose p = 1, w = 2 and r = 3 the profit maximizing monopolist will produce*
    a) *0 units*
    b) *2 units*
    c) *10 units*
    d) *None of the above*

24. *Consider an exchange economy with persons 1 and 2 and goods x and y. Person 1's utility function is* $u_1(x, y) = x + y$ *and* $u_2(x, y) = 4x^2 + 4y^2 + 8xy$. *The total endowment of the economy is (2, 1). Which of the following is true about this economy?*
    a) *All feasible allocations are efficient*
    b) *Only allocations on the diagonal from 1's origin to 2's origin is efficient*
    c) *Only allocations on the four boundaries of the edgeworth box are efficient*
    d) *None of the above*

25. *Consider the situation of the preceding question. If person 1's endowment is (1, 1) and 2's endowment is (1, 0), then the following allocation is a competitive equilibrium*

a) $(x_1, y_1) = (1.5, 0.2)$ $(x_2, y_2) = (0.5, 0.8)$
b) $(x_1, y_1) = (1.8, 0.2)$ $(x_2, y_2) = (0.2, 0.8)$
c) $(x_1, y_1) = (1, 0.5)$ $(x_2, y_2) = (1, 0.5)$
d) $(x_1, y_1) = (1.1, 0.6)$ $(x_2, y_2) = (0.9, 0.4)$

26. *Consider the situation of the preceding question. Which of the following is an equilibrium price vector?*
    a) $(p_x, p_y) = (8/13, 5/13)$
    b) $(p_x, p_y) = (1/2, 1/2)$
    c) $(p_x, p_y) = (4/5, 1/5)$
    d) $(p_x, p_y) = (1, 0)$

(Problems 27 to 28) Consider market of a good with the following characteristic: There are three consumers of the good. Set of consumers is denoted by {1, 2, 3}. Each consumer wants to "consume" at most one unit of the good. Good can be bought only in non-negative integer quantities. Consumer i's valuation of the first unit of the good is given by v_i where $v_1 = 2$, $v_2 = 4$, $v_3 = 7$. Given price p, consumer i's demand correspondence is the following:

$x_i(p) = 1$ *if* $v_i - p \geq 0$

$= 0$ *if* $v_i - p < 0$

i.e. consumer will demand the unit for consumption if his valuation for it is greater than or equal to the price. Clearly, market demand function is given by $x(p) = \sum_i x_i(p)$

Now consider a monopolist producing the good at cost c per unit.

27. *Suppose c = 0, the profit maximizing monopolist will produce*
    a) 0
    b) 1
    c) 2
    d) 3

28. *The value of c for which the profit maximizing monopolist produces either 1 or 2 units*
    a) 1
    b) 2
    c) 3
    d) 4

29. *Your teacher knows 5 jokes and in each class tells 2 jokes; each joke has an equal chance of being selected. What is the probability that, in a given lecture, at least 1 joke is told that was not told in the previous class?*
    a) 9/10
    b) 8/10
    c) 7/10
    d) 6/10

30. *Consider the following claim: " There is some university and some department such that all the faculty members of that department are noble laureates." If this claim is false, then which of the following statements must be true?*
    a) *In every university, there exists a department such that no faculty member in it is a noble laureate*
    b) *There is some university and some department such that no faculty member of that department is a noble laureate*
    c) *In every university, every department no faculty member is a noble laureate*
    d) *In every university, every department has at least one faculty member who is not a noble laureate*

31. *A competitive industry faces the demand* $x = 865 - 5p$
    Where x is the industry output and p is the price of the product. There are n firms having identical cost conditions $C_i = 144 + 5x_i + 4x_i^2$ *where* x_i *is the output of firm i and* C_i *is its total cost. There is free entry and large number of potential entrants.*
    Equilibrium output and price of the industry is
    a) $(x, p) = (690, 35)$
    b) $(x, p) = (600, 53)$
    c) $(x, p) = (515, 70)$
    d) *None of the above*

32. *In the preceding question, number of firms who will enter the industry is*
    a) *115*
    b) *100*
    c) *90*
    d) *None of the above*

(Problems 33 to 38) Consider a macro economy where the aggregate output in the short run is given by:

$Y = (\min \{L, K^*\})^{\alpha}$ *, where L is the total employment of labor;*

K^* *is the total capital stock (which is fixed in the short run);*

$0 < \alpha < 1$*is the parameter of the system*

Let P and W denote the aggregate price level and the money wage rate respectively. Assume that the producers in the economy maximize profit in the perfectly competitive set up.

33. *The corresponding demand for labor schedule as a function of the real wage is given by:*
    a) $L^d = \min\{(w/(\alpha P))^{1/(\alpha-1)}, K^*\}$
    b) $L^d = \max\{(w/(\alpha P))^{1/(\alpha-1)}, K^*\}$
    c) $L^d = (w/(\alpha P))^{1/(\alpha-1)}$
    d) $L^d = K^*$

34. *If there is a one shot increase in the stock of the capital (K*), the demand for labor schedule (labor demand on horizontal axis and real wage on vertical axis), as derived above,*
    a) *shifts leftwards completely*
    b) *shifts rightwards completely*
    c) *shifts leftwards with some part of the curve coincides with the original curve*
    d) *shifts rightwards with some part of the curve coincides with the original curve*

Suppose the above economy is characterized by a single household which takes the aggregate price level and the money wage rate as given and decides on its consumption and labor supply by maximizing its utility subject to its budget constraint. The household has a total endowment of L units of labor time, of which it supplies L^s units to the market at the money wage rate W, and enjoys the rest as leisure. Its utility depends on its consumption and leisure in the following way: $U = C (L^* - L^s)$ The only source of income of the household is the wage income and it spend its entire wage earning in buying consumption goods at the price P.*

35. *The corresponding supply of labor schedule as a function of the real wage is given by:*
    a) $L^s = L^*$
    b) $L^s = L^*/3$
    c) $L^s = (W/P)L^*$
    d) $L^s = L^*/2$

36. *If there is an exogenous increase in the total endowment of labor time (L*), the supply of labor schedule, as derived above,*
    a) *shifts right*
    b) *shifts left*
    c) *does not shift*
    d) *the information available is not adequate to answer this question*

37. *Let $\alpha = ½$, $K^* = 100$, $L^* = 8$. Given these parameter values, the unique non-negative equilibrium value of real wage rate that clears the labor market is given by:*
    a) $W/P = ¼$
    b) $W/P = ½$
    c) $W/P = ¾$
    d) *None of the above*

38. *Consider the labor demand and the labor supply schedule with parameter values given in problem 37, the aggregate supply curve (output supplied as a function of the aggregate price level) is*
    a) *upward sloping*
    b) *downward sloping*
    c) *vertical*
    d) *horizontal*

39. *The following equations describe an economy. Desired consumption and investment are*

C = 0.8(1 – t)Y
I = 900 – 50i
G = 800
t = 0.25
Money demand is $M^d/P = 0.25Y – 62.5i$
Real Money Supply M/P = 500
Level of output and interest rate at which both money market and goods market clear equals
a) (Y, i) = (3500, 6)
b) (Y, i) = (3000, 4)
c) (Y, i) = (4000, 8)
d) (Y, i) = (4000, 2)

40. *Consider a consumer with utility function u(x, y) = min{2x, x+y}. Price of x is Rs. 50 and price of y is Rs. 100. Income of the individual is Rs. 300. In equilibrium, consumer will consume*
    a) (2, 2)
    b) (6, 0)
    c) (0, 3)
    d) None of the above