Practice Questions:

1. Three coworkers would like to know their average salary. However, they are self-conscious and don't want to tell each other their own salaries, for fear of either being ridiculed or getting their houses robbed. How can they find their average salary, without disclosing their own salaries?

2. In addition shown below, each of the letters A, B, C, D, and E represents one of the ciphers from 1 up to 5 (equal letters represent equal ciphers and different letters represent different ciphers). The first and last ciphers of the sum are given.

```
   ABCDE
   DABEC
   EAABC
 + ACDAE
   -----
   9CBA0
```

   The Question: What does the complete addition look like in ciphers?

3. In Josephine's Kingdom every woman has to take a logic exam before being allowed to marry. Every married woman knows about the fidelity of every man in the Kingdom *except* for her own husband, and etiquette demands that no woman should tell another about the fidelity of her husband. A gunshot fired in any house in the Kingdom will be heard in any other. Queen Josephine announced that unfaithful men had been discovered in the Kingdom, and that any woman knowing her husband to be unfaithful was required to shoot him at midnight following the day she discovered his infidelity. How did the wives manage this?

4. Each letter represent a different number. Find the numbers such that the addition below is satisfied:

```
     S E N D
 +   M O R E
 = M O N E Y
```

5. It's an old saying that "two wrongs don't make a right." However here is one case where the proverb is not quite true. For although it is undeniably correct to say that in the two subtractions

```
N I N E                 N I N E
- T E N               -  O N E
----------              ----------
  T W O                  A L L
```

   we have two wrongs, nevertheless when the digits which the

various letters represent are correctly identified the two calculations will be found to be absolutely right.
What number must each letter represent in order that these two subtractions may simultaneously be decoded into correct computations?

6. Clark, Daw and Fuller make their living as carpenter, painter and plumber, though not necessarily respectively. The painter recently tried to get the carpenter to do some work for him, but was told that the carpenter was out doing some remodelling for the plumber. The plumber makes more money than the painter. Daw makes more money than Clark. Fuller had never heard of Daw. What is each man's occupation?

7. Many merchants adopt the policy of marking their price tags in a code in which each digit from zero to nine is represented by a different letter. This enables the salesman to tell at a glance the price of an article and at the same time keeps the customers in ignorance of the cost until the clerks reveal it.
So habitual was this practice with a certain Mr. Pythagoras J. Countinghouse that he began to do all his calculation with letters instead of with numbers, to the great discomfiture of his bookkeepers. Such a system is much too simple to remain long a secret, however, and one day a customer found a scrap of paper in the store with the following computation on it, and in almost no time at all had figured out just which letter stood for each number.

```
E C A ) F D B H J ( A B J
        C G G
        -------
        A G A H
        A A E A
       ----------
          K D D J
          K D B H
         ---------
              A J
```

8. Dorothy, Jean, Virginia, Bill, Jim and Tom are six young persons who have been close friends from their childhood. They went through high school and college together, and when they finally paired off and became engaged nothing would do but a triple announcement party. Naturally they wanted to break the news to

their friends in an unusual fashion, and after some thought they decided upon this scheme.
At just the right moment during the party everyone was given a card bearing the cryptic information:

> Who now are six will soon be three,
> And gaily we confess it,
> But how we've chosen you may know
> No sooner than you guess it.

Tom, who is older than Jim, is Dorothy's brother. Virginia is the oldest girl. The total age of each couple-to-be is the same although no two of us are the same age. Jim and Jean are together as old as Bill and Dorothy.
What three engagements were announced at the party?

9. Inspite of appearances the following is not a spelling lesson but a multiplication problem in which each digit has been replaced by a different letter.

```
            F  U  R
            D  O  G
    ----------------
         A  G  E  R
      D  R  I  P
      F  U  R
    ----------------
      R  I  P  E  R
```

What two numbers were multiplied?

10. In a certain problem in long division every digit except 7 was suppressed, yielding

```
                . 7 .
        ______________
  . .  | . . . . . .
         . 7 7
         -----
           . 7 .
           . 7 .
           -----
               . .
               . .
               ---
```

Restore the missing digits. I hope you would find this puzzle challenging.

11. Consider the following coded subtraction:

```
   E  I  G  H  T
-  T  H  R  E  E
---------------------
      F  I  V  E
---------------------
```

If each letter is supposed to stand for a different digit, prove that there is no possible way to assign a unique digit to each letter to form a correct subtraction.

12. In a certain multiplication problem each digit from 0 to 9 was replaced by a different letter, yielding the coded calculation

```
                A  L  E
                R  U  M
     ------------------------------
            W   I  N  E
         W  U   W  L
      E  W  W   E
  -------------------------------------
      E  R  M   P  N   E
```

For what number does each letter stand?

13. You are given the following information:
1) If A is P, C is not R.
2) If B is P or R, A is Q.
3) If A is Q or R, B is P.

Determine the correspondence between the symbols (A, B, C) and the symbols (P, Q, R).

14. Shorty Finelli was found shot to death one morning, and the police with better than average luck had three red-hot suspects behind bars by nightfall. That evening the men were questioned and made the following statements.

Buck: 1) I didn't do it.
2) I never saw Joey before.
3) Surely, I knew Shorty.

Joey: 1) I didn't do it.
2) Buck and Tippy are both pals of mine.
3) Buck never killed anybody.

Tippy: 1) I didn't do it.

2) Buck lied when he said he had never seen Joey before.
3) I don't know who did it.

If one and only one of each man's statement is false, and if one of the three men is actually guilty, who is the murderer?

15. Ann, Ben, Can, and Den think of one natural number.

Ann says it consists of two digits.
Ben says it is a divisor of 150.
Can says it is not 150.
Den says it is divisible by 25.

Which one of them is not telling the truth?

16. A painting job can be completed by 7 painters in 41 days. If 21 more painters join the team 9 days after starting work on the job, then how many more days are required to complete the job?

17. Alice
Betty
Cindy
Debbie

Mrs. Harrison has 6 lovely hats, 3 blue, 2 yellow and one pink. Alice, Betty, Cindy and Debbie are lined up as shown in the figure. Mrs. Harrison help them put the hats on them so they will not see what color hat they have on.

Alice can see what color of hats Betty, Cindy and Debbie are wearing. Betty can see what color of hats Cindy and Debbie are wearing. Cindy can see what color hat Debbie is wearing. Debbie can not see any of the hat colors.

Mrs. Harrison ask them what color of the hat they are wearing. Alice said she can not tell. Betty said she can not tell either. Cindy also can not tell. However, Debbie was able to tell what color she was wearing after knowing that everyone else could not tell.

How did Debbie figure out what color she was wearing?

Solutions:

1. Let 3 coworkers be A, B and C. A tells B and not to C that, "if my salary is $a (may or may not be right), then what is the average between two of us" B replies to C and not to A after computing the average based on his own true salary and the salary told by A, "Our average salary is $b=($a+$true salary of B)/2". Then C computes the overall average, say $c=(2*($b)+$true salary of C)/3. And C says to A and not to B,"Our average salary is $c". Then A announces the true average.

2.
```
    25431
    32514
    11254
  + 24321
    -----
    94520
```

3. If there is only 1 unfaithful husband, then every woman in the Kingdom knows that except for his wife, who believes that everyone is faithful. Thus, as soon as she hears from the Queen that unfaithful men exist, she knows her husband must be unfaithful, and shoots him.
   If there are 2 unfaithful husbands, then both their wives believe there is only 1 unfaithful husband (the other one). Thus, they will expect that the case above will apply, and that the other husband's wife will shoot him at midnight on the next day. When no gunshot is heard, they will realize that the case above did *not* apply, thus there must be more than 1 unfaithful husband and (since they know that everyone else is faithful) the extra one must be their own husband.
   If there are 3 unfaithful husbands, each of their wives believes there to be only 2, so they will expect that the case above will apply and both husbands will be shot on the second day. When they hear no gunshot, they will realize that the case above did *not* apply, thus there must be more than 2 unfaithful husbands and as before their own husband is the only candidate to be the extra one.
   In general, if there are n unfaithful husbands, each of their wives will believe there to be $n-1$ and will expect to hear a gunshot at midnight on the $n-1$th day. When they don't, they know their own husband was the nth.

4. The solution to this puzzle is O=0, M=1, Y=2, E=5, N=6, D=7, R=8, and S=9.

5. N 1
   W 2
   I 3
   A 5
   T 6
   O 8
   E 9
   L 0

6. Clark is the carpenter, Daw the painter, and Fuller the plumber.

7. A 2
   B 6
   C 7
   D 8
   E 3
   F 9
   G 4
   H 0
   J 5
   K 1

8. The party announced the engagements of Dorothy to Jim, Jean to Tom, and Virginia to Bill.

9. 1 D
   2 A
   3 F
   4 O
   5 R
   6 E
   7 G
   8 I
   9 U
   0 P

10. The obvious point to attack here is the first partial product, _77, since it is the most nearly determined number in the problem. Now the only one digit numbers whose product ends in 7 are 3 and 9. Hence the first digit in the quotient must be one of these numbers and the last digit in the divisor must be the other. If we consider the possible divisors of the form _9 and multiply each by 3, we find that the only one which yields the product of the form _77 is 59 which gives 177. Alternatively, if we try the divisors of the form _3 and multiply each by 9 we find that only 53 yields

a product of the form _77. We must reject the first of these two possibilities, however, since when 59 is multiplied by the second digit in the quotient, namely 7, the result is 413, whereas according to the problem the second partial product is of the form _7_. This leaves 53 as the unique possibility for the divisor and 9 as the first digit of the quotient. Finally we observe that the last digit of the quotient must be 1 since the last partial product contains just two digits. Knowing that the divisor is 53 and the quotient is 971, we can multiply these numbers to obtain the dividend. The rest of the problem can then be reconstructed at once.

11. We notice first that in the leftmost column the subtraction of T from E leaves 0. Hence E must be exactly one more than the T (the 1 having been borrowed from E for use in the second column). Now in the rightmost column, T minus E yields E. (Since E is greater than T, 1 had to be borrowed from the column on the left to make this subtraction possible.) Or to put it in the reverse sense, E plus E is a two digit number having T in the units place. Hence T must be even, and ofcourse different from 0 since it appears as the leftmost digit in the second row of the problem. We therefore have the following possibilities:
    T: 2 4 6 8
    E: 6 7 8 9
    Among these there is only one pair, namely E = 9, T = 8, which meets the further requirement that E is 1 more than T.
    Now consider the subtraction in the second column from the right. We have already observed that 1 had to be borrowed from the H for use in the column on the right. Hence E, that is 9, taken away from 1 less than H leaves V. But first borrowing 1 from a number and then taking 9 away from what remains is clearly just the same as taking 10 away from the original number.
    And when 10 is subtracted from any number, the units digit of the number necessarily appears unchanged as the units digit of the answer. Hence the result of the subtraction in the second column from the right must be H and cannot be the different digit V. This inescapable contradiction proves that the problem cannot be decoded to produce a correct subtraction.

12. To systematize our work we first write in a row the different letters appearing in the problem:
    A L E R U M W I N P

    Over each letter we will write its numerical equivalent when we discover it. In the columns under the various letters we will record clues and tentative hypothesis, being careful to put all related inferences on the same horizontal line.

In problems of this sort the digit 0 and 1 can often be found, or atleast restricted to a very few possibilities, by simple inspection. For instance, 0 can never occur as the leftmost digit of an integer, and when any number is multiplied by zero the result consists exclusively of zeros. Moreover, when any number is multiplied by 1 the result is that number itself. In the present problem, however, we can identify 0 by an even simpler observation. For in the second column from the right, N plus L equals N, with nothing carried over from the column on the right. Hence L must be 0.

In our search for 1 we can eliminate R, U and M at once, since none of these, as multipliers in the second row, reproduces A L E. Moreover E cannnot be 1 since U times E does not yield a product ending in U. At present, however, we have no further clues as to whether 1 is A, I, N, P or W.

Now the partial product W U W L ends in L, which we know to be 0. Hence one of the two letters U and E must be 5. Looking at the units digits of the other partial products, we see that both M X E and R X E are numbers ending in E. A moment's reflection show that (or a glance at a multiplication table) shows that E must therefore be 5.

But if E is 5, then both R and M must be odd, since an even number multiplied by 5 would yield a product ending in 0, which is not the case in either the first or third partial product. Moreover by similar reasoning it is clear that U is an even number.

At this point it is convenient to return to our array and list under U the various possibilities, namely 2,4,6 and 8. Opposite each of these we record the corresponding value of W as read from the partial product W U W L, whose last two digits are now determined since the factor A L E is known to be _ 0 5. These values of W are easily seen to be 1,2,3 and 4.

From an inspection of the second column from the left we can now deduce the corresponding possibilities for R. As we have already noted R must be odd; hence its value is twice W plus 1 (the one being necessarily carried over from the column on the right). The possible values for R are then 3, 5, 7 and 9, and our array looks like this:

```
  0 5
A L E R U M W I N P
      3 2   1
      5 4   2
      7 6   3
      9 8   4
```

Now in the third column from the left in the example the sum of the digits W, U and W must be more than 9, since 1 had to be carried over from this column into the column on the left. The values in the first two rows of the array are too low for this, however, hence we can cross out both of these lines.

A further consideration of the sum of the digits W, U and W in the third column from the left, coupled with the fact that M is known to be odd, shows that in the third row of the array M must be 3 while in the fourth row it must be 7. This permits us to reject the third row of array also, for it contains 3 for both M and W, which is impossible. The correct solution must therefore be the one contained in the forth row. Hence R is 9, U is 8, M is 7, and W is 4. Substituting these into the problem it is a simple matter to determine that A is 6, I is 2, N is 3, and P is 1. This completes the solution.

6	0	5	9	8	7	4	2	3	1
A	L	E	R	U	M	W	I	N	P

13. A is Q
    B is P
    C is R

14. Joey is the murderer.

15. Den is lying. Think about the natural number in question now.

16. 8 more days are required

17. The only situation that Alice can tell what color she was wearing was that nobody was wearing blue hat. In this situation, she must had wearing a blue hat. We can be sure one of the girls in front of Alice was wearing blue hat.

    Betty knew the above statement was true. If neither Cindy nor Debbie was not wearing blue hat, then she can be sure she was wearing blue hat. Obviously she saw a blue hat in front of her.

This was why she could not tell.

Cindy had the same situation as Betty. She cold not tell because she saw a blue hat in front of her.

Debbie knew all the above facts. So she told Mrs. Harrison that she was wearing a blue hat.