There are 14 problems in the document. These problems are selected from the forum and my emails to facilitate your preparation for JNU entrance exam. Each problem starts from a new page for your convenience. All the best.

1. In the multiplication problem a, b, c, d, e, f, g, j stand for distinct integers; 0 stands for zero. Find values and provide explanation:

$$\begin{array}{r} a\ b\ c\ d\ 9 \\ \underline{\quad\ f\ 4\ g} \\ j\ a\ 4\ d\ 4 \\ c\ 4\ b\ a\ g \\ \underline{f\ d\ a\ c\ j\_\_\_\_} \\ b\ b\ 4\ 0\ 4\ b\ 4 \end{array}$$

Solution:

a 1

b 3

c 5

d 7

f 2

g 6

j 8

2. Four students a, b, c, d competed in mathematics, logic, literature and economics contests and each one of them won one contest. Below are given the forecasts made by these students:
   A: d will win the logic contest
   B: c will win the economics contest
   C: a will not win the mathematics contest
   D: b will win the literature contest
   It turned out that the forecasts made by the winners of the literature and economics contests were wrong; and those made by the winners of the mathematics and logic contests were right. Who won which contest?

Solution:

Mathematics–D; Logic–C; Literature–B; Economics–A

3. Suppose there are 4 alternatives x,y,z and w. Further suppose that there are 7 individuals 1,2,3,4,5,6 and 7. The individuals rankings(orderings) of the four alternatives R_i, i=1,.....7 are given by
   R_1:(xy)zw
   R_2:yzwx
   R_3:zw(xy)
   R_4:(xy)(zw)
   R_5:yzwx
   R_6:zw(xy)
   R_7:(xy)(zw)
   Determine which of the alternatives are pareto Optimal. Expalin your answer.
   (Alternatives inside the parentheses are indifferent to each other. If an alternative is written to the left of another alternative ,then the former is preffered to the latter)

Solution:

y and z are pareto efficient, x and w are not pareto efficient

4. Use the derivative of the function y = square root of x to obtain an appropriate value for square root of 16.02

Solution:

$y = \sqrt{x}$

$dy/dx = (1/2\sqrt{x})$

To compute the square root of 16.02, we do the following:

$\sqrt{16.02} \approx \sqrt{16} + (1/2\sqrt{16})*(0.02) = 4 + 0.0025 = 4.0025$

5. X and Y are independent random variables. Let V_1 and V_2 denote the variance of X and Y, respectively. Let the random variable Z be defined to be [aX+bY] , where a and b are two positive real numbers. It is known that $V_2>1$, variance of Z is 1, and covariance between Y and Z is 1. In terms of V_1 and V_2, what are a and b?

Solution:

$V(Z) = 1$ implies $a^2V_1+b^2V_2 = 1$

$Cov(Y, Z)= Cov(Y, aX+bY) = bV_2 = 1$

So, $b = 1/V_2$ and $a = \sqrt{((V_2-1)/ V_1V_2)}$

6. The house numbers of 3 individuals: A,B and C are,;1)distinct from each other, 2)lie between 1 and 99.
   Both facts are known to all 3 individuals. B asks A when C is not present,2 questions: 1) Is your house number a perfect square? 2) Is your house number greater than 50? Assuming A's answers to be correct, B is able to infer house number of A from answers given by A.A's house number, however is not the same as inferred by B because A answered only second question truthfully. C asks A when B is not present, 2 questions:1)Is your house number a perfect cube?2) Is your house number greater than 25? Assuming A's answer to be correct, C is able to infer house number of A from answer to two questions given by A. A's house number, however is not same as inferred by C because A answered only second question truthfully. Deduce the house numbers of 3 individuals given that:1) A's house number is less than house number of both B and C. 2) sum of house numbers of 3 individuals is perfect square multiplied by two.

Solution:

A's House Number is 55
B's House Number is 81
C's House Number is 64

Working:

Given that, B asks A when C is not present, 2 questions:
1) Is your house number a perfect square?
2) Is your house number greater than 50?
Assuming A's answers to be correct, B is able to infer house number of A from answers given by A.
Since B is able to infer house number of A from answers given by A the only possibility is that A has answered in 'yes' to both the questions. So that there are two possibilities: 64 and 81. And He can infer the house number of A only if his own house number is one of these two numbers.

Given that, C asks A when B is not present, 2 questions:
1) Is your house number a perfect cube?
2) Is your house number greater than 25?
Assuming A's answers to be correct, C is able to infer house number of A from answers given by A.
Since C is able to infer house number of A from answers given by A the only possibility is that A has answered in 'yes' to both the

questions(since A answered 'yes' in reply to B's second question, so he must answer 'yes' to C's second question). So that there are two possibilities: 27 and 64. And He can infer the house number of A only if his own house number is one of these two numbers.

Given that A's house number is less than B and C's house number and A's house number is greater than 50, this implies that C's house number is 64 and B's house number is 81.

For getting house number of A, we will first use the following information:
Sum of house numbers of 3 individuals is perfect square multiplied by two.
Given that B's House Number is 81, C's House Number is 64, let A's house number be x. Also we know that 100 > x > 50. Now, x+81+64 is a perfect square multiplied by 2. This implies that (x+81+64)/2 = (x + 145)/2 is a perfect square. Since 100 > x > 50 this implies that 245/2 > (x + 145)/2 > 195/2. Now we look for a perfect square in the interval (195/2, 245/2)
=(97.5, 122.5)
Two Possibilities are 100, 121
Solving, (x + 145)/2 = 100 give us x = 55
and (x + 145)/2 = 121 give us x = 97
Now using the condition that A's house number is less than house number of both B and C we rule out x = 97
Hence, A's house number is 55.

7. Four persons A, B, C and D have to share Rs.4 among themselves in units of one rupee. First A proposes a distribution and all of them, including A ,vote on it. if at least 50%of those voting agree with A, the proposal is accepted. if not, A loses her voting rights and B gets to propose a distribution and all except A vote on it. once again, B's proposal is accepted if at least 50% of those eligible to vote agree on it. if not,B also loses her voting rights and C gets to propose and so on D.
assume that each person prefers more money to less, and will always vote against a distribution in which she gets zero. What distribution should A propose in the beginning?

Solution:

A offers 3 to himself, 1 to C and 0 to B and D.

Working:

Let's do the backward induction argument,
When only C and D are left, C will offer 4 to himself and 0 to D and since C's vote carry 50% weight, hence the offer will be agreed.
When B, C and D are left, C will reject any offer that give him less than 4 and D will accept any offer greater than 0 since this is what they are going to get if B is eliminated. So, B will offer 3 to himself and 1 to D and the offer will be accepted by 2/3rd majority.
Thus, C will get 0 if B, C and D are left.
So, when its A's turn to make an offer, C will accept any offer greater than 0. So, A will make an offer equal to 3 to himself and 1 to C and the offer will be accepted by 50% votes.

8. For a race, 5 individuals made forecasts. Each of them made 2 statements, atleast one of which turned out to be false. Determine who won which medal.

   Statements by individual 1: A will not win the gold medal. B will not win the silver medal.
   Statements by individual 2: C will win a medal. D will not win a medal.
   Statements by individual 3: D will win a medal. E will win a medal.
   Statements by individual 4: D will not win the silver medal. E will not win the bronze medal.
   Statements by individual 5: A will win a medal. C will not win a medal.

Solution:

Since atleast one of the statements made by each individual is false.
We can figure out who wins the medals by doing something like this: Suppose A wins a medal, by (5) this implies that C must also win some medal, by (2) this inturn implies that D must win a medal, now by (3) this implies that E must not win a medal. So, by (4) D must win the silver medal. Now by (1) we have A must win the gold medal. Thus, C wins the bronze medal.
So, A wins gold, D wins silver and C wins bronze.

9. There are 100 competitive firms in an industry, each with a short run cost curve $C(q)=q^2+4q+K$, where K is a positive constant. The industry dd curve is $Q=100-10p$. Find the short run equilibrium for this industry. What limit must K satisfy for the firms to survive, all other things remaining the same?

Solution:

$p = MC = 2q + 4$ is the supply curve of each firm

so $q = p/2 - 2$. With 100 firms, market supply curve is

$Q = 100(p/2 - 2) = 50p-200$

Using industry demand curve, $Q=100-10p$

We can now solve for equilibrium, $p = 5$

$Q = 50$, so $q = 0.5$

For firms to survive in the long run

$AC \leq p$

$K \leq 0.25$

10. Suppose we say that an allocation x is pareto optimal iff there does not exist an allocation, say y, such that: every individual in the economy considers y to be at least as good as x, and at least one individual considers y to be strictly better than x. Now consider a society of 4 individuals. this society has five allocations to choose from a, b, c, d and e. Individual preferences over these allocations are as follows:
    1st individual: a P_1 b P_1 c P_1 d P_1 e
    2nd individual: b P_2 c P_2 d P_2 e P_2 a
    3rd individual: c P_3 d P_3 e P_3 a P_3 b
    4th individual: d P_4 e P_4 a P_4 b P_4 c
    a P_1 b P_1 c P_1 d P_1 e means that the 1st individual prefers allocation a over b, b over c....d over e. Likewise for the preferences of other individuals. Find out pareto optimal allocations for this society.

Solution:

a, b , c, d are pareto efficient.

11. A foreign institutional investor who will settle only for returns in excess of 10 percent, is considering converting dollars into rupees to purchase a stock that is valued at Rs.100 but is expected to appreciate in value to Rs.110 at the end of one year. If the investing entity expects the rupee to depreciate in value from Rs. 50 to a dollar to Rs.52.50 to a dollar in that year, it would purchase the stock only if the firm is expected to pay a dividend of:
    a] Rs.3 per share
    b] Rs.2.50 per share
    c] Rs.5 per share
    d] more than Rs. 5 per share

Solution:

Investors invest $2 to buy one stock/share. Price of the stock in Indian currency(50 X 2) is Rs. 100.
The minimum value it expects to get after an year is $2.20. The minimum return in indian currency(52.5 X 2.20) is Rs. 115.50
Since the value of stock is going to be Rs. 110, the firm will invest only if it is paid a minimum dividend of Rs. 5.50 (115.50-110)
Hence, the answer is (d)

12. A chocolate bar has been stolen by either child a or b. One of child is innocent. Here are the statements of witnesses and other interested parties
    c: a has not stolen the chocolate bar
    d: a has stolen things in the past
    e: b has stolen things in the past
    f: e has stolen things in the past
    g: d and e are both right
    h: d and f are both right
    i: e or f is right , and may be both
    j: g or h is right , and may be both
    k: d and i are right
    l: j is right and k is wrong

    Suppose that c and l are either both telling the truth or both lying. Who stole the chocolate bar?

Solution:

Assumption: Suppose c and l are telling the truth.
This implies a has not stolen the chocolate bar. i.e. a is innocent.
And j is right and k is wrong.
j is right implies g or h is right, and may be both
k is wrong implies d or i is wrong or may be both
g or h is right, and may be both implies d is right and e or f is right, and may be both.
d or i is wrong or may be both implies d is wrong or both e and f are wrong.
So we arrive at a contradiction.
Hence our assumption is false.
Thus, c and l are lying. This implies that a has stolen the chocolate bar.

13. One of a, b, c and d has committed a crime with the help of another one of them. Here are the statements by the four individuals. The statements of the criminal and his accomplice are false and those of the remaining two are true.
a: if b is guilty of something , then c must be innocent .
b: if a is innocent then c must be guilty
c: if b was the killer then d must have had nothing to do with the crime
d: i am innocent

Which of the four is the criminal and who is his accomplice?

Solution:

The simplest way could be to consider all six cases one by one and eliminate wrong ones one by one till you find the correct one:
i) A and B are innocent--> C and D are lying, C is lying implies B was killer and D is his accomplice, contradicting B is innocent
ii)A and C are innocent--> B and D are lying, B is lying implies A and C are innocent, that's fine, D is lying implies he is not innocent, that is fine too, A is telling the truth because B is guilty and C is innocent, C is telling the truth implies that B was not the killer which inturn imples that D was the killer and B was his accomplice.
So, we have found out that D was killer and B was accomplice.
We can rule out other cases if we want to:
iii) A and D are innocent---> A is telling the truth and B is guilty implies C is innocent, contradicting C is guilty
iv) B and C are innocent--> A is lying which implies that B and C are guilty, contradicting B and C are innocent.
v) B and D are innocent--> A is lying which implies that B and C are guilty, contradicting B is innocent.
vi) C and D are innocent--> A is lying which implies that B and C are guilty, contradicting C is innocent.

So, A and C were innocent, D was killer and B was his accomplice.

14. There were 4 candidates namely a, b, c, d for an award

(1) only one of the four candidates had a distinction both in maths and literature.
(2) only one candidate had a distinction both in maths and philosophy.
(3) only one candidate had a distinction both in maths and history.
(4) only one candidate had a distinction both in literature and philosophy.
(5) only one candidate had a distinction both in literature and history.
(6) only one candidate had a distinction both in philosophy and history.
(7) both a and b had a distinction in maths.
(8) both c and d had a distinction in literature.
(9) both b and c had a distinction in philosophy.
(10) d had a distinction in history.

The award was given to the candidate who had distinction in more than one subject than any other candidate, which candidate was given the award?

Solution:

Following is the right answer (Working is little long, hence omitted):
a has distinctions in Mathematics and Literature only.
b has distinctions in Mathematics, Philosophy and History only.
c has distinctions in Literature and Philosophy only.
d has distinctions in Literature and History only.

Hence, b is given the award.