2013

Test Code: QEA /QEB (Both Descriptive types)
(Junior Research Fellowship in Quantitative Economics)

The candidates for Junior Research Fellowship in Quantitative Economics are required to take two descriptive tests - QEA (Mathematics) in the forenoon session and QEB (Economics) in the afternoon session.

Syllabus for QEA

1. Permutations and combinations.
2. Elementary set theory; Functions and relations; Matrices.
3. Functions of one and several variables: limits, continuity, differentiation, applications, integration of elementary functions, definite integrals, theory of quadratic equations.
4. Constrained and unconstrained optimization, convexity of sets and concavity and convexity of functions.
5. Elements of probability theory, discrete and continuous random variables, expectation and variance, joint conditional and marginal distributions, distributions of functions of random variable.

Syllabus for QEB

1. Theory of consumer behaviour; theory of production; market structure; general equilibrium and welfare economics; international trade and finance; public economics.
2. Macroeconomic theories of income determination, Rational Expectations, Phillips Curve, Neo-classical Growth Model, Inequality.
3. Game Theory: Normal and extensive forms, Nash and sub-game perfect equilibrium.
4. Statistical inference, regression analysis (including heteroscedasticity, autocorrelation and multicollinearity), least squares and maximum likelihood estimation, specification bias, endogeneity and exogeneity, instrumental variables.

Sample Questions for QEA

1. Let S be a set with n elements. Use the fact that
$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n,$$
and *Pascal's rule*:
$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$
to show that the number of subsets of S that have an even number of elements is 2^{n-1}.

2. It is sufficient to give formulae for the answers below without calculating the exact answers.

   (a) (10 marks) Leela has 6 friends. Each evening, for 5 days, she invites 3 of them for dinner so that exactly the same group is never invited more than once. In how many ways could she do this?

   (b) (10 marks) How many of the non-negative integers from 0 to 9999 have no two adjacent digits the same?

3. For any (possibly infinite) set A, and any $B \subset A$, if there is an injection (1-1 function) $f : A \to B$, then there is also a bijection (1-1 correspondence) $h : A \to B$. Use this fact to prove the *Schroeder-Bernstein Theorem*: If S and T are sets, and there exist injections $f : S \to T$ and $g : T \to S$, then there exists a bijection $h : S \to T$.

4. Let $f : \mathbb{R} \to \mathbb{R}$ ($\mathbb{R}$ is the set of real numbers) be defined as follows. For every $t \in \mathbb{R}$,
$$f(t) = \begin{cases} -1 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0. \end{cases}$$
Define the following function $g : [-2, 2] \to \mathbb{R}$ as follows. For every $x \in [-2, 2]$,
$$g(x) = \int_{-2}^{x} f(t)dt.$$

(a) (15 marks) Plot the function g.

(b) (5 marks) Using this plot (or otherwise) find $\lim_{x\to 0} g(x)$.

5. Let $f(x) = xg(x)$, where g is a real-valued continuous function defined on the interval $[-1, 1]$. Prove that f is differentiable at 0 and find its derivative in terms of g.

6. A function $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is said to be *quasiconcave* if all its *upper contour sets* $\{x \in \mathbb{R}^n : f(x) \geq y\}, y \in \mathbb{R}$, are convex, or equivalently, if it is the case that for all $\lambda \in [0, 1], f(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{f(x_1), f(x_2)\}$. Suppose $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is quasiconcave and $g : \mathbb{R} \longrightarrow \mathbb{R}$ is non-decreasing. Prove that $F : \mathbb{R}^n \longrightarrow \mathbb{R}$, defined by $F(\mathbf{x}) = g[f(\mathbf{x})]$ is quasiconcave.

7. Consider the following utility maximization problem of a consumer:

$$\max_{x_1\geq 0,\ x_2\geq 0} \log(x_1 + a) + \log(x_2 + b) \text{ subject to } p_1x_1 + p_2x_2 \leq M$$

where $a > 0$ and $b > 0$ are constants. As usual, p_1 and p_2 are the prices of goods 1 and 2 respectively and M represents the consumer's income.

Write down the necessary conditions for a maximum. Find the solution in all cases along with the corresponding restrictions on the parameters a, b, p_1, p_2, and M.

8. A slip of paper is given to person A, who marks it with either a plus or minus sign; the probability of her writing a plus sign is $\frac{1}{3}$. A passes the slip to B, who may either leave it alone or change the sign before passing it to C. Finally, C passes the slip to a referee after perhaps changing the sign. The referee sees a plus sign on the slip. It is known that B and C each change the sign with probability $\frac{2}{3}$. Compute the probability that A originally wrote a plus sign.

9. Let X be a random variable with PDF (probability density function)

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad -\infty < x < \infty.$$

Find the PDF of X^2.

Sample Questions for QEB

1. Consider the following fixed proportions (Leontief) technology

$$Y = F(K, L) = \min(AK, BL)$$

where $A > 0$ and $B > 0$ are constants, Y is output, K is aggregate capital, and L is the labor force.

   (a) Derive the intensive form production function, $y = f(k)$, where $y = Y/L$ and $k = K/L$. Plot the intensive form production function in y, k space. Under what condition is capital fully employed ?

   (b) Suppose the capital accumulation equation is given by

$$\frac{\dot{k}}{k} = s\frac{f(k)}{k} - (n + \delta)$$

   where $n > 0$ is the population growth rate, and $\delta \in [0, 1]$ is the exogenous depreciation rate of capital. Under what condition will there be no positive steady state value of k (call this steady state, $\bar{k}$)? What happens to $\frac{\dot{k}}{k}$ under this condition? What happens to the level of unemployment under this condition? Draw a figure to explain your answer.

   (c) Under what condition is there a positive steady state value of k? Does the steady state now feature unemployed workers ? How about idle machines ?

   (d) What is the only way to reach a steady state in which all capital and labor is employed ? Is there any reason for this condition to hold ? Explain your answer in a few sentences.

2. The government is debating a replacement of the current public distribution system with either a direct cash transfer or a food coupon program. The food coupon program will provide below poverty line families with coupons which can be exchanged for some specified rupee value worth of food at any grocery store. A cash transfer program

will provide the same rupee value of cash as in the coupons program, directly to such families. Many economists argue that an equivalent cash subsidy would bring about a greater increase in the well-being or utility of low income families receiving aid. Advocates of the food coupon program argue that this would provide more incentives for low income families to increase food consumption. Graphically analyze the two sides of the arguments with respect to the following (assume usual convex shaped indifference curves):

(a) Increases in consumer well-being under cash transfers and food coupons program.

(b) Increase in food consumption under cash transfers and food coupons program.

3. Consider a market comprising 100 customers. All of them are willing to pay Rs. 1500 for one unit of a product, X. Half of them are willing to pay Rs. 2100 for one unit of another product, Y; the other half are willing to pay at most zero for one unit of Y. Customers want at most one unit of any product. Further, customers buy either X or Y; thus, the consumers who are willing to pay Rs. 2100 for one unit of Y do not want X in case they buy Y. There is a monopoly firm that can produce X at a cost of Rs. 1000 per unit. Moreover, it can transform one unit of X into 1 unit of Y at a cost of 500. The firm does not know the type of consumers, i.e., who are willing to pay a positive amount for Y. (If a group of consumers is indifferent between buying two products or between buying a product and not buying it, assume that half of them choose each option).

   (a) Solve for the firm's profit maximizing strategy if it can produce both X and Y.[Prices have to be in whole Rupees]

   (b) Suppose consumers can buy one unit of X from the firm and transform it into 1 unit of Y using a backyard technology at a per unit cost of Rs. 610. Is the same strategy still profit maximizing? Why?

4. Consider two countries, A and B. Each country has 120 total labour hours to produce two goods X and Y, and each country has preferences given by the utility function

$$U = \min\{x, y\}.$$

No other inputs are required to produce these goods. Their production technologies are the following. In country A, production of one unit of X requires 2 labour hours and that of Y requires 1 labour hour. In country B, production of one unit of X requires 1 labour hour and that of Y requires 2 labour hours.

   (a) Find the equilibrium allocation of labour hours between the goods in each country under a no trade situation.

   (b) Now suppose that each country has specialized in the activity in which it has a comparative advantage and then they trade at a 'one for one' price. Find the optimal consumption of two goods by each country.

   (c) Draw the production possibility frontier of each country and that of the joint production possibility frontier explaining their shapes and slopes.

5. The 8-7 plan, an effort of the Chinese government to reduce rural poverty, began in 1994. Counties (or districts) were designated eligible for the program if, in 1994, they had less than 400 yuan per capita income. Eligible counties received three benefits under this plan: subsidized loans for rural enterprise, budgetary funds and funds for public works projects. Suppose you have county-level data on per capita income, and other county characteristics for 1994 (at baseline or just before the plan started) and the same data ten years later in 2004. Consider the estimating equation below:

$$Y_{it} = b_0 + b_1 T_i + b_2 D_t + b_3 T_i D_t + u_{it},$$

where Y_{it} is the per-capita income in county i at time t; T_i is a dummy for treatment status of county i (=1 if eligible and 0 otherwise); D_t is

a dummy variable equal to 1 if $t = 2004$, and 0 if $t = 1994$; and u_{it} is the error term.

(a) Interpret the coefficients b_0, b_1, b_2, and b_3 saying what they represent.

(b) Do you think this regression model gives us a consistent estimate of the program's impact on poverty? Under what assumptions? Explain.

6. Consider the model,

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

where $E(\epsilon|x_1) = E(\epsilon|x_2) = E(\epsilon|x_1, x_2) = 0$, and $E(\epsilon^2|x_1) = E(\epsilon^2|x_2) = E(\epsilon^2|x_1, x_2) = \sigma^2$. Suppose a researcher collects data on the relevant variables and estimates the model by ordinary least squares. Later a very wise professor points out that β_2 is actually zero. Does this information matter? In particular, should the researcher drop the variable x_2?

7. Two agents want to split an amount one unit of a divisible good. Each agent $i = 1, 2$ announces a non-negative real number x_i. Both agents make their announcements simultaneously. Each agent i pays an amount of the good equal to his announcement, i.e., x_i units. If $x_i > x_j$ $(i, j \in \{1, 2\})$ then agent i gets the entire 1 unit of the good and agent j receives nothing. If $x_1 = x_2$, then each agent receives $\frac{1}{2}$ a unit. The net utility of each agent is the amount of the good they receive minus the amount they announce. A pure strategy for any agent in this game is a non-negative real number. Does this game have a pure strategy Nash equilibrium? Explain your answer.

8. Consider an economy consisting of a continuum 1 of dynasties. Each individual lives for two periods, childhood and adulthood, and gives birth to one child in his adulthood. There is consumption only at the end of adulthood. Preferences are

$$(1 - \delta) \log c_i(t) + \delta \log e_i(t)$$

where c_i is consumption at the end of individual i's life, and e_i is the educational spending on the child of this individual. The credit market does not exist so that there is no borrowing or lending and each individual's budget constraint is

$$c_i(t) + e_i(t) \leq w_i(t),$$

where w_i denotes the wage income of individual, i. Assume that the labour market is competitive, and the wage income of individual i is simply a linear function of his human capital:

$$w_i(t) = Ah_i(t)$$

where $A > 0$. The human capital of the child of individual i of generation t in turn is given by

$$h_i(t+1) = \begin{bmatrix} e_i(t)^\gamma \text{ if } e_i(t) \geqslant 1 \\ \\ \bar{h} \qquad \text{if } e_i(t) < 1 \end{bmatrix}$$

where $\gamma \in (0,1)$ and $\bar{h} \in (0,1)$ is some minimum level of human capital that the individual attains even without any educational spending. Each individual chooses the spending on education that maximizes his own utility. This implies the following educational spending on the child of an individual:

$$e_i(t) = \delta w_i(t) = \delta A h_i(t)$$

Assume that: $\delta A > 1 > \delta A\bar{h}$.

(a) Describe the dynamics of human capital for a particular dynasty i explaining the history dependence clearly. Illustrate the dynamics by plotting $h_i(t)$ on the x-axis and $h_i(t+1)$ on the y-axis. Are there multiple steady states? Are there multiple equilibria? Explain clearly.

(b) Explain the presence of poverty traps in the above economy.

9. Let X_1 and X_2 be a random sample from a population with the following population density

$$f_x(x) = \begin{bmatrix} \frac{1}{a}, & \text{if } x = 0 \\ \\ 1 - \frac{1}{a}, & \text{if } x = 1 \end{bmatrix}$$

(a) Compute the likelihood function, $L(a; X_1, X_2)$ and report your results based on the following table

$L(a; X_1, X_2)$	$X_2 = 0$	$X_2 = 1$
$X_1 = 0$		
$X_1 = 1$		

(b) Assume that the realization of the random sample is $X_1 = 0$ and $X_2 = 1$. Compute the maximum likelihood estimators of a and a^2?

(c) Assume that the realization of the random sample is $X_1 = 1$ and $X_2 = 1$.What happens to the maximum likelihood estimator of a?

(d) Assume that the realization of the random sample is $X_1 = 0$ and $X_2 = 1$. What is the method of moment estimator of a, using the first population moment.