2016

Test Codes: QEA and QEB (Both are of 'Descriptive' type)

(Junior Research Fellowship in Quantitative Economics)

The candidates for Junior Research Fellowship in Quantitative Economics are required to take two descriptive-type tests - QEA in the forenoon session and QEB in the afternoon session. While the questions in QEA are on Mathematics, those in QEB are on Economics.

Syllabus for QEA

1. Permutations and combinations.
2. Theory of quadratic and cubic equations; Elementary set theory.
3. Matrix algebra, rank and inverse of matrices, linear equations, determinants, and eigenvalues.
4. Functions of one and two variables: Limits, continuity, differentiation, applications, integration of elementary functions, and definite integrals.
5. Constrained and unconstrained optimization, convexity of sets ,and concavity and convexity of functions.
6. Elements of probability theory, discrete and continuous random variables, expectation and variance, joint conditional and marginal distributions, and distribution of function of a random variable.

1. (a) If $|a| < 1$ and $|b| < 1$, then find if the series

$$a(a+b) + a^2(a^2+b^2) + a^3(a^3+b^3) + \ldots\ldots$$

converges or not.

(b) Let x_1 and x_2 be the roots of the quadratic equation $x^2 - 3x + a = 0$ and x_3 and x_4 be the roots of the quadratic equation $x^2 - 12x + b = 0$. If x_1, x_2, x_3 and x_4 $(0 < x_1 < x_2 < x_3 < x_4)$ are in G.P., then find the value of ab.

2. A bivariate probability density function is defined by

$$f_{X,Y}(x, y) = C(x+2y) \text{ if } 0 < y < 1 \text{ and } 0 < x < 2$$
$$= 0 \text{ otherwise}$$

where C is a constant.

(a) Find the value of C.
(b) Find the marginal distribution of X.
(c) Find the joint cumulative distribution function of X and Y.

3. For the matrix $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, show that there does not exist any invertible matrix Q and a diagonal matrix D such that $Q^{-1}A\,Q = D$.

4. Let $f(x)$ = minimum $\{x, 10 - x\}$, $x \geq 0$. For any non-negative real number t, let $x(t)$ be the (global) maxima of $f(x)$ for $x \in [0, t]$. Find the function $x(t)$.

5. Find all positive solutions of the following system of equations:

$$x_1 + x_2 = x_3^2,$$
$$x_2 + x_3 = x_4^2,$$
$$x_3 + x_4 = x_5^2,$$
$$x_4 + x_5 = x_1^2,$$

and $$x_5 + x_1 = x_2^2.$$

6. Suppose a real-valued function f over $[0,\infty)$, satisfies the following properties: (a) $f(x)$ is continuous for $x \geq 0$, (b) $f'(x)$ exists for $x > 0$, (c) $f(0) = 0$, and (d) $f'(x)$ is monotonically increasing.
Now define another real-valued function g over $(0,\infty)$, as $g(x) = \dfrac{f(x)}{x}$ for $x > 0$.
Show that $g(x)$ is a monotonically increasing function.

7. Let f be a real valued function defined on 2×2 real matrices $A = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ where $a_1 = (a_{11}, a_{12})'$ and $a_2 = (a_{21}, a_{22})'$ are any two real 2-dimensional row vectors. Further, f satisfies the following properties.

(i) f is a linear function of each row when the other row is held fixed. For example, when the second row is held fixed,

$$f\begin{pmatrix} \delta\, a_1 + a_1^* \\ a_2 \end{pmatrix} = \delta\, f\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + f\begin{pmatrix} a_1^* \\ a_2 \end{pmatrix}$$

where a_1^* is any real 2-dimensional row vector and δ is any real number.

(ii) $f\begin{pmatrix} a_1 \\ a_1 \end{pmatrix} = 0$ for all a_1.

(iii) $f\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 1$ when $a_1 = (1,0)$ and $a_2 = (0,1)$

Show that f is the determinant of A.

8. A deck of 52 playing cards is shuffled, and the cards are turned up one at a time until the first 'ace' appears. Find which of the following two events A and B is more likely to happen.
A: The next card drawn (i.e., the card following the first 'ace') is the 'ace' of spades; and
B: The next card drawn is the '2' of clubs.

9. Let $f: \Re \rightarrow \Re$ be a twice differentiable function such that $f(\xi_i) = 0$ for $i = 1, 2, 3$ where ξ_i's are distinct. Show that the second derivative of f vanishes at a point.

10. Find the value of the integral

$$\int_A x^2 e^{xy} dxdy$$

where A is the region bounded by the straight lines $y = x$, $y = 0$, and $x = 1$.

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Syllabus for QEB

1. Theory of consumer behaviour; Theory of production; Market structure; General equilibrium and welfare economics; International trade and finance; Public economics.
2. Macroeconomic theories of income determination, rational expectations, Phillips Curve, neo-classical growth model, and inequality.
3. Game Theory: Normal and extensive forms, and Nash and sub-game perfect equilibrium.
4. Multiple Regression Model: Least squares and maximum likelihood methods of estimation, heteroscedasticity, autocorrelation , multicollinearity, specification bias, exogeneity, and instrumental variables.
5. Time Series Analysis: Stationarity, trend, seasonality, unit root tests, and ARMA model.

1. A multinational firm is to decide whether it will export to a host country or open a 100% owned subsidiary in the host country. In case of export it faces a tariff $t > 0$ per unit for its exports, and in case of subsidiary formation it has to incur a set up cost, $F > 0$. In either case the MNC will have to compete with a host firm. The marginal costs of production of the MNC and the host firm are respectively, c_m and c_h where $c_m < c_h$. The market demand function in inverted form is given by $P = a - Q$ where P is the price of the product and Q is the industry output and a (>0) is a constant. Further assume that the MNC cannot be monopoly in the labour market even if the tariff rate be zero.

How do you decide the optimal entry strategy of the MNC? Let $\hat{t}(F)$ be the locus of (F, t) for which the MNC is indifferent between these two modes of entry. Obtain the sign of $\frac{d\hat{t}}{dF}$. Explain how this locus is helpful to decide on the entry strategy of the MNC.

If the market demand function be downward sloping and strictly concave, what can you say about the sign of $\frac{d\hat{t}}{dF}$?

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2. In a research and development project there are decreasing returns to spending money faster. The more rapidly a given sum is spent, the less it contributes to total effective effort. Let $x(t)$ be the rate of spending at t and $z(t)$ be the cumulative effective effort devoted to the project by time t. Assume

$$\dot{Z}(t) = [x(t)]^{1/2}$$

where a 'dot' on a variable denotes derivative with respect to time. The total effective effort required to complete the project is A. Hence we have

$$Z(0) = 0, \quad Z(T) = A$$

where T is the time when the project is completed and is to be determined from the optimization exercise. Assume a reward R can be collected when the project is completed and the rate of discount is r.

(a) Write down the problem of the firm undertaking the project to maximize the net present value of return from the project.

(b) Find the optimal time path of $x(t)$ and $Z(t)$.

(c) Find the optimal terminal time T. How does T vary with the reward R? Explain your answer.

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3. (a) An economy saves and invests 45% of its national income and its labour force grows at 5% rate. The production function satisfies constant returns to scale, and it is also Cobb-Douglas and symmetric in terms of its arguments. Find out the steady-state equilibrium value of capital labour ratio following the one- sector growth model of Solow.

(b) Following an exogenous change in money supply, the AD curve (aggregate demand in the complete Keynesian model) is found to shift horizontally by 10 units. The slopes of the IS and LM curves are given to be - 0.004 and 0.004 respectively. Compute the horizontal shift of the LM curves, following the aforesaid change in money supply.

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4. Consider an economy in which policy makers are concerned about both inflation and unemployment, with a loss function given by

$$L = \frac{1}{2}\,(y - \bar{y})^2 + \frac{\theta}{2}\,(\Pi - \overline{\Pi})^2 \;, \quad \theta > 0$$

where y = current input, $\bar{y}$ = desired output, Π = actual inflation, and $\overline{\Pi}$ the desired inflation rate. The coefficient 'θ' measures the relative importance of the deviation of inflation from its desired /target value in the loss function.

The economy is characterized by an expectation augmented Phillips Curve given by

$$y = y_L + \alpha\ (\Pi - \Pi^e) + U,\ \alpha > 0,\ y_L < \tilde{y}$$

where U is the disturbance term with zero mean and constant variance, Π^e = expected inflation, and y_L is the long-run level of output.

(a) Suppose the policy maker announces a binding commitment to low inflation. What is the value of inflation that minimizes the expected value of the loss function L?

(b) Obtain the equilibrium level of inflation under discretion.

(c) Under which case - discretion or commitment - is inflation higher?

Explain your answer.

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5. Consider a village with n households, $n \geq 2$. Each household owns a pump-set, which it uses to pump up ground water for irrigation purposes. The monetary benefit to a household, i, from using w_i amount of water is given by the increasing, concave and twice differentiable function $B(w_i)$. There is however some probability, p, that the water pumped up will be contaminated with arsenic. This probability increases, at an increasing rate, as the water table falls. The water table falls with an increase in total consumption of water in the village. The monetary cost incurred by a household, in case a member contracts arsenic poisoning, is c. Suppose that households are risk-neutral expected utility maximizers, and choose their water consumption simultaneously.

(a) Model this problem as a normal form game. Show that the level of arsenic poisoning will be excessive in the Nash equilibrium, generating a sub-optimal outcome.

(b) Suppose now that the village is split into two religious communities, A and B; λ being the proportion of the population that belongs to community A. Each individual internalizes the cost to all other members of her own community, but not to members of the other community. Show how the level of arsenic poisoning changes with the extent of homogeneity in the village population (i.e., with the value of λ).

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6. There are n individuals who witness a crime. Everybody would like the police to be called. If this happens, each individual derives satisfaction $v > 0$ from it. Calling the police has a cost of c, where $0 < c < v$. The police will come if at least one person calls. Hence, this is an n-person game in which each player chooses from $\{C, N\}$; C means 'call the police' and N means 'do not call the police'. The payoff to person i is 0 if nobody calls the police, $v - c$ if i (and perhaps others) call the police, and v if the police is called but not by person i.

(a) What are the Nash equilibria of this game in pure strategies? In particular, show that the game does not have a symmetric Nash equilibrium in pure strategies (a Nash equilibrium is symmetric if every player plays the same strategy).

(b) Compute the symmetric Nash equilibrium or equilibria in mixed strategies.

(c) For the Nash equilibrium/equilibria in (b), compute the probability of the crime being reported. What happens to this probability if n becomes large?

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7. Consider a consumer in a two-good economy whose commodity space is $\Re^2_+$. Assume that the consumer's preference is complete and continuous. Then answer the following questions.

(a) If the preference relation is monotonic, then show that for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ such that $x_i \geq y_i \ \forall \ i \in \{1,2\}$, x is at least as good as y.

(b) Assume that preference relation is strictly monotonic and $p = (p_1, p_2)$ is a price vector such that $p_i \geq 0 \ \forall \ i \in \{1,2\}$ and $(p_1, p_2) \neq (0,0)$. Suppose that $x^* = (x_1^*, x_2^*)$ is a consumption bundle such that $x_i^* > 0 \ \forall \ i \in \{1,2\}$ and $p.x^* \leq I$, where I denotes the income of the consumer. It is also given that for any y strictly preferred to x, we have $p.y \geq I$. Then show that $p_1 > 0$ and $p_2 > 0$.

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8. (a) In order to obtain an earning function, income of a labour is regressed on his educational level. Three educational levels are considered. The regression model

$$y = \alpha_0 + \alpha_1 E_1 + \alpha_2 E_2 + u$$

is estimated by the ordinary least squares (OLS) method, where E_1 and E_2 are two dummy variables indicating membership in the first and second educational levels. Show that the OLS estimates of α_0, α_1 and α_2 i.e., $\hat{\alpha}_0, \hat{\alpha}_1$ and $\hat{\alpha}_2$, are

$$\begin{bmatrix} \hat{\alpha}_0 \\ \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{bmatrix} = \begin{bmatrix} \bar{y}_3 \\ \bar{y}_1 - \bar{y}_3 \\ \bar{y}_2 - \bar{y}_3 \end{bmatrix}$$

where $\bar{y}_i$ denotes the mean value of y in the i^{th} educational level.

(b) The usual two-variable linear regression model is postulated and a sample of 20 observations is drawn from an urban area and another sample of 10 observations is drawn from a rural area. The sample information in raw form is summarized as follows:

Urban $\quad X'X = \begin{bmatrix} 20 & 20 \\ 20 & 25 \end{bmatrix}, \quad X'y = \begin{bmatrix} 10 \\ 20 \end{bmatrix}, \quad y'y = 30$

Rural $\quad X'X = \begin{bmatrix} 10 & 10 \\ 10 & 20 \end{bmatrix}, \quad X'y = \begin{bmatrix} 8 \\ 20 \end{bmatrix}, \quad y'y = 24$

where y is the vector of observations on the dependent variable and X is the matrix of observations on the independent variables. Test the hypothesis that the same relationship holds in both and urban areas.

Note that the critical value of F (2, 26) at 5% level of significance is 3.37.

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9. (a) A researcher estimates the following two econometric models

$$y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + u_i$$

and
$$y_i = \beta_1^* + \beta_2^* x_{2i} + \beta_3^* x_{3i} + \beta_4^* x_{4i} + v_i$$

where u_i and v_i are independently and identically distributed disturbances, and x_3 is an irrelevant variable which does not enter the actual data generating process. Would the value of (i) R^2 and (ii) adjusted R^2, be higher for the second model than the first? Explain your answers.

(b) Suppose a stationary time series $\{y_t\}$ follows the following model:

$$y_t = \Phi y_{t-1} + u_t \;, \qquad t = 1, 2, \ldots, n$$
$$u_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} \;, \; \varepsilon_t \sim \text{white noise } \left(0, \sigma_\varepsilon^2\right).$$

Show that the ordinary least squares estimation is not applicable for estimating the parameters of the model. Obtain an appropriate method of estimation stating additional assumptions, if any, which would produce consistent estimators of the parameters. Justify, with derivations wherever necessary, why this method is appropriate.

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10. (a) State clearly what you understand by partial autocorrelation function (PACF) of a time series. Show that for an autoregressive process of order 2, $\Phi_{11} \neq 0$, $\Phi_{22} \neq 0$ but $\Phi_{kk} = 0$ for $k \geq 3$ where Φ_{kk} stands for the k^{th} partial autocorrelation of the time series.

(b) Consider the following two models under the null (H_0) and alternative (H_1) hypotheses:

$$H_0 : \alpha = 1 \text{ in } y_t = \alpha\, y_{t-1} + u_t$$

$$H_1 : |\alpha| < 1 \text{ in } y_t - \delta = \alpha(y_{t-1} - \delta) + u_t$$

where $u_t \sim$ white noise $(0, \sigma^2)$ and δ is assumed to be very large. Show that the unit root test based on the ordinary least squares estimator, $\hat{\alpha}$, from the equation under H_o has low power against H_1 i.e., $\hat{\alpha} \to 1$ under H_1, in large samples.

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11. The National Rural Employment Guarantee Act (NREGA) was passed in 2005, implemented in 200 backward districts from February 2006, and extended to all districts from April 2008. The aim was to provide employment to rural households in public works when they could not easily find other employment. Using survey data from the two years 2005 and 2007 for all districts in India, a researcher is interested in estimating the effect of the NREGA on rural wages *Y*.

(a) She first estimates the following regression by OLS using data only from 2007:

$$Y_i = \alpha_0 + \alpha_1 N_i + u_i,$$

where *i* denotes a district, *N* is a dummy variable for one of the backward districts in which the NREGA was implemented first, and *u* is the disturbance term. Will the estimate of α_1 be an unbiased estimate of the impact of the NREGA on rural wages? If not, what is the likely direction of the bias? Explain.

(b) She next estimates the following regression using data from both years, but only for the districts that received the NREGA in 2006.

$$Y_{it} = \beta_0 + \beta_1 D_{it} + v_{it}$$

where *t* denotes one of the years 2005 and 2007, D = 1 if *t* = 2007 and zero otherwise, and *v* is the disturbance term. Will the estimate of β_1 be an unbiased estimate of the impact of the NREGA on rural wages? If not, what is the likely direction of the bias? Explain.

(c) She next estimates the following regression using data from both years and all districts.

$$Y_{it} = \gamma_0 + \gamma_1 D_{it} + \gamma_2 N_{it} + \gamma_3 D_{it} N_{it} + w_{it} \ ,$$

where w is the disturbance term. What do the parameters $\gamma_0, \gamma_0 + \gamma_1$, and $\gamma_0 + \gamma_2$ represent in this regression model? What would you expect the signs of γ_1 and γ_2 to be ? Under what assumption will the estimate of γ_3 be an unbiased estimate of the impact of the NREGA on rural wages? What does the assumption mean in economic terms?

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