SYLLABUS AND SAMPLE QUESTIONS FOR MSQE
(Program Code: MQEK and MQED)
2015
Syllabus for PEA (Mathematics), 2015

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations; (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants. Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency, dispersion, correlation and regression, probability distributions, standard distributions-Binomial and Normal.

Sample questions for PEA (Mathematics), 2015

1. $\lim_{x \to 0+} \frac{\sin\{\sqrt{x}\}}{\{\sqrt{x}\}}$, where $\{x\}$ = decimalpart of x, is

   (a) 0 (b) 1 (c) non-existent (d)none of these

2. $f : [0,\ 1] \to [0,\ 1]$ is continuous.Then it is true that

   (a) $f(0) = 0,\ f(1) = 1$
   (b) f is differentiable only at $x = \frac{1}{2}$
   (c) $f'(x)$ is constant for all $x \in (0,1)$
   (d) $f(x) = x$ for at least one $x \in [0,1]$

3. $f(x) = |x-2| + |x-4|$.Then f is
   (a) continuously differentiable at x = 2
   (b) differentiable but not continuously differentiable at $x = 2$
   (c) f has both left and right derivatives at $x = 2$
   (d) none of these

4. In an examination of 100 students, 70 passed in Mathematics, 65 passed in Physics and 55 passed in Chemistry. Out of these students, 35 passed in all the three subjects, 50 passed in Mathematics and Physics, 45 passed in Mathematics and Chemistry and 40 passed in Physics and Chemistry.Then the number of students who passed in exactly one subject is

(a) 30 (b) 25 (c) 10 (d) none of these

5. The square matrix of the matrix $\begin{vmatrix} a & b \\ c & 0 \end{vmatrix}$ is a null matrix if and only if

(a) $a = b = c = 0$
(b) $a = c = 0$, b is any non-zero real number
(c) $a = b = 0$, c is any non-zero real number
(d) $a = 0$ and either $b = 0$ or $c = 0$

6. If the positive numbers x, y, z are in harmonic progression, then
$\log(x + z) + \log(x - 2y + z)$ equals

(a) $4\log(x - z)$ (b) $3\log(x - z)$
(c) $2\log(x - z)$ (d) $\log(x - z)$

7. If f(x+2y, x-2y)= xy, then f(x,y) equals

(a) $\dfrac{x^2 - y^2}{8}$ (b) $\dfrac{x^2 - y^2}{4}$ (c) $\dfrac{x^2 + y^2}{4}$ (d) none of these

8. The domain of the function $f(x) = \sqrt{x^2 - 1} - \log\left(\sqrt{1 - x}\right)$, $x \geq 0$, is

(a) $(-\infty, -1)$ (b) $(-1, 0)$ (c) null set (d) none of these

9. The graph of the function $y = \log(1 - 2x + x^2)$ intersects the x axis at

(a) 0, 2 (b) 0,- 2 (c) 2 (d) 0

10. The sum of two positive integers is 100.The minimum value of the sum of their reciprocals is

(a) $\dfrac{3}{25}$ (b) $\dfrac{6}{25}$ (c) $\dfrac{1}{25}$ (d) none of these

11. The range of the function $f(x) = 4^x + 2^x + 4^{-x} + 2^{-x} + 3$, where $x \in (-\infty, \infty)$, is

(a) $\left(\frac{3}{4}, \infty\right)$ (b) $\left[\frac{3}{4}, \infty\right)$ (c) $(7, \infty)$ (d) $[7, \infty)$

12. The function $f : R \to R$ satisfies $f(x+y) = f(x) + f(y) \forall x, y \in R$, where R is the real line, and $f(1) = 7$ .Then $\sum_{r=1}^{n} f(r)$ equals

(a) $\frac{7n}{2}$ (b) $\frac{7(n+1)}{2}$ (c) $\frac{7n(n+1)}{2}$ (d) $7n(n+1)$

13. Let f and g be differentiable functions for $0 < x < 1$ and $f(0) = g(0) = 0, f(1) = 6$ .Suppose that for all $x \in (0, 1)$, the equality $f'(x) = 2g'(x)$ holds. Then $g(1)$ equals

(a) 1 (b) 3 (c) -2 (d) -1

14. Consider the real valued function $f(x) = ax^2 + bx + c$ defined on [1, 2].Then it is always possible to get a $k \in (1, 2)$ such that

(a) $k = 2a + b$ (b) $k = a + 2b$ (c) $k = 3a + b$ (d) none of these

15. In a sequence the first term is $\frac{1}{3}$.The second term equals the first term divided by 1 more than the first term. The third term equals the second term divided by 1 more than the second term, and so on. Then the 500th term is

(a) $\frac{1}{503}$ (b) $\frac{1}{501}$ (c) $\frac{1}{502}$ (d) none of these

16. In how many ways can three persons, each throwing a single die once, make a score of 10?

(a) 6 (b) 18 (c) 27 (d) 36

17. Let a be a positive integer greater than 2.The number of values of x for which $\int_a^x (x + y)dy = 0$ holds is

(a) 1 (b) 2 (c) a (d) a + 1

18. Let (x^*, y^*) be a solution to any optimization problem $max_{(x,y)\in\Re^2} f(x, y)$ subject to $g_1(x, y) \leq c_1$. Let (x', y') be a solution to the same optimization problem $max_{(x,y)\in\Re^2} f(x, y)$ subject to $g_1(x, y) \leq c_1$ with an added constraint that $g_2(x, y) \leq c_2$.Then which one of the following statements is always true?

(a) $f(x^*, y^*) \geq f(x', y')$ (b) $f(x^*, y^*) \leq f(x', y')$
(c) $|f(x^*, y^*)| \geq |f(x', y')|$ (d) $|f(x^*, y^*)| \leq |f(x', y')|$

19. Let (x^*, y^*) be a real solution to: $max_{(x,y)\in\Re^2} \sqrt{x} + y$ subject to $px + y \leq m$, where $m > 0, p > 0$ and $y^* \in (0, m)$.Then which one of the following statements is true?

(a) x^*depends only on p (b) x^* depends only on m
(c) x^* depends on both p and m (d) x^* is independent of both p and m.

20. Let $0 < a_1 < a_2 < 1$ and let $f(x; a_1, a_2) = -|x - a_1| - |x - a_2|$.Let X be the set of all values of x for which$f(x; a_1, a_2)$ achieves its maximum. Then

(a) $X = \{x|x \in \left\{\frac{a_1}{2}, \frac{1+a_2}{2}\right\}\}$ (b) $X = \{x|x \in \{a_1, a_2\}\}$
(c) $X = \{x|x \in \left\{0, \frac{a_1+a_2}{2}, 1\right\}\}$ (d) $X = \{x|x \in [a_1, a_2]\}$.

21. Let A and B be two events with positive probability each, defined on the same sample space. Find the correct answer:

(a) P(A/B) > P(A) always (b) P(A/B) < P(A) always
(c) P(A/B) > P(B) always (d) None of the above

22. Let A and B be two mutually exclusive events with positive probability each, defined on the same sample space. Find the correct answer:

(b) A and B are necessarily independent
(c) A and B are necessarily dependent
(d) A and B are necessarily equally likely
(e) None of the above

23. The salaries of 16 players of a football club are given below (units are in thousands of rupees).

100, 100, 111, 114, 165, 210, 225, 225, 230,
575, 1200, 1900, 2100, 2100, 2650, 3300

Now suppose each player received an extra Rs. 200,000 as bonus. Find the correct answer:
(a) Mean will increase by Rs. 200,000 but the median will not change
(b) Both mean and median will be increased by Rs. 200,000
(c) Mean and standard deviation will both be changed
(d) Standard deviation will be increased but the median will be unchanged

24. Let Pr(X=2)= 1. Define $\mu_{2n} = E(X-\mu)^{2n}, \mu = E(X)$. Then:
(a) μ_{2n}=2 (b) μ_{2n}=0 (c) μ_{2n}> 0 (d) None of the above

25. Consider a positively skewed distribution. Find the correct answer on the position of the mean and the median:

(a) Mean is greater than median (b) Mean is smaller than median
(c) Mean and median are same (d) None of the above

26. Puja and Priya play a fair game (i.e. winning probability is ½ for both players) repeatedly for one rupee per game. If originally Puja has *a* rupees and Priya has *b* rupees (where a>b), what is Puja's chances of winning all of Priya's money, assuming the play goes on until one person has lost all her money?

(a) 1 (b) 0 (c) b/(a+b) (d) a/(a+b)

27. An urn contains *w* white balls and *b* black balls (*w>0*) and (*b>0*).The balls are thoroughly mixed and two are drawn, one after the other, *without* replacement. Let *Wi* denote the outcome 'white on the *i*-th draw' for *i*=1,2.Which one of the following is true?
(a) P(W2) = P(W1) = w/(w+b)
(b) P(W2) = P(W1) = (w-1)/(w+b-1)
(c) P(W1)= w/(w+b), P(W2)= (w-1)/(w+b-1)
(d) P(W1)= w/(w+b), P(W2) = {w(w-1)}/{(w-b)(w+b-1)}

28. A bag contains four pieces of paper, each labeled with one of the digits 1, 2, 3, 4, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3?

(a) ¾ (b) ½ (c) ¼ (d) 9/24

29. Consider two random variables X and Y where X takes values -2,-1,0,1,2 each with probability 1/5 and Y=|X|. Which of the following is true?
    (a) The variables X and Yare independent and Pearson's correlation coefficient between X and Y is 0.
    (b) The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 0.
    (c) The variables X and Y are independent and Pearson's correlation coefficient between X and Y is 1.
    (d) The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 1.
30. Two friends who take the metro to their jobs from the same station arrive to the station uniformly randomly between 7 and 7:20 in the morning. They are willing to wait for one another for 5 minutes, after which they take a train whether together or alone. What is the probability of their meeting at the station?

    (a) 5/20 (b) 25/400 (c) 10/20 (d) 7/16

Syllabus for PEB (Economics), 2015

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample questions for PEB (Economics), 2015

1. Consider an agent in an economy with two goods X_1 and X_2.Suppose she has income 20.Suppose also that when she consumes amounts x_1 and x_2 of the two goods respectively, she gets utility

$$u(x_1, x_2) = 2x_1 + 32x_2 - 3{x_2}^2.$$

    (a) Suppose the prices of X_1 and X_2 are each 1.What is the agent's optimal consumption bundle? [**5 marks**]
    (b) Suppose the price of X_2 increases to 4, all else remaining the same.Which consumption bundle does the agent choose now? [**5 marks**]
    (c) How much extra income must the agent be given to compensate her for the increase in price of X_2? [**10 marks**]

2. Suppose a government agency has a monopoly in the provision of internet connections. The marginal cost of providing internet connections is ½, whereas the inverse demand function is given by: $p = 1 - q$. The official charge per connection is set at 0; thus, the state provides a subsidy of ½ per connection. However, the state can only provide budgetary support for the supply of 0.4 units, which it raises through taxes on consumers. Bureaucrats in charge of sanctioning internet connections are in a position to ask for bribes, and consumers are willing to pay them in order to get connections. Bureaucrats cannot, however, increase supply beyond 0.4 units.

   (a) Find the equilibrium bribe rate per connection and the social surplus. [**5 marks**]

   (b) Now suppose the government agency is privatized and the market is deregulated; however, due large fixed costs of entry relative to demand, the privatized company continues to maintain its monopoly. Find the new equilibrium price, bribe rate and social surplus, specifying whether privatization increases or reduces them. [**10 marks**]

   (c) Suppose now a technological innovation becomes available to the privatized monopoly, which reduces its marginal cost of providing an internet connection to c, $0 < c < ½$.Find the range of values of c for which privatization increases consumers' surplus. [**5 marks**]

3. Suppose the borders of a state, B, coincide with the circumference of a circle of radius r > 0, and its population is distributed uniformly within its borders (so that the proportion of the population living within some region of B is simply the proportion of the state's total land mass contained in that region), with total population normalized to 1.For any resident of B, the cost of travelling a distance d is kd, with $k>0$.Every resident of B is endowed with an income of 10, and is willing to spend up to this amount to consume one unit of a good, G, which is imported from outside the state at zero transport cost. The Finance Minister of B has imposed an entry tax at the rate $100t\%$ on shipments of G brought into B. Thus, a unit of G costs $p(1+t)$ inside the borders of B, but can be purchased for just p outside; $p(1+t) < 10$.Individual residents of B have to decide whether to travel beyond its borders to consume the good or to purchase it inside the state. Individuals can travel anywhere to shop and consume, but have to return to their place of origin afterwards.

   (a) Find the proportion of the population of B which will evade the entry tax by shopping outside the state. [**5 marks**]

   (b) Find the social welfare-maximizing tax rate. Also find the necessary and sufficient conditions for it to be identical to the revenue-maximizing tax rate. [**5 marks**]

   (c) Assume that the revenue-maximizing tax rate is initially positive. Find the elasticity of tax revenue with respect to the external price of G, supposing that the Finance Minister always chooses the revenue-maximizing tax rate. [**10 marks**]

4. Suppose there are two firms, 1 and 2, each producing chocolate, at 0 marginal cost. However, one firm's product is not identical to the product of the other. The inverse demand functions are as follows:

$$p_1 = A_1 - b_{11}q_1 - b_{12}q_2, p_2 = A_2 - b_{21}q_1 - b_{22}q_2;$$

where p_1 and q_1 are respectively price obtained and quantity produced by firm 1 and p_2 and $q_2$are respectively price obtained and quantity produced by firm 2.$A_1, A_2, b_{11}, b_{12}, b_{21}, b_{22}$are all positive.Assume the firms choose independently how much to produce.

   (a) How much do the two firms produce, assuming both produce positive quantities? [**10 marks**]

   (b) What conditions on the parameters $A_1, A_2, b_{11}, b_{12}, b_{21}, b_{22}$ are together both necessary and sufficient to ensure that both firms produce positive quantities? [**5 marks**]

   (c) Under what set of conditions on these parameters does this model reduce to the standard Cournot model? [**5 marks**]

5. Suppose a firm manufactures a good with labor as the only input. Its production function is $Q = L$, where Q is output and L is total labor input employed. Suppose further that the firm is a monopolist in the product market and a monopsonist in the labor market. Workers may be male (M) or female (F); thus, $L = L_M + L_F$.Let the inverse demand function for output and the supply functions for gender-specific labor be respectively $p = A - \frac{Q}{2}$; $L_i = {w_i}^{\varepsilon_i}$, $\varepsilon_i > 0$; where p is the price received per unit of the good and w_i is the wage the firm pays to each unit of labor of gender i, $i \in \{M, F\}$.Let $\varepsilon_M \varepsilon_F = 1$.Suppose, in equilibrium, the firm is observed to hire both M and F workers, but pay M workers double the wage rate that it pays F workers.
   (a) Derive the exact numerical value of the elasticity of supply of male labor. [**10 marks**]
   (b) What happens to total *male* labor income as a proportion of total labor income when the output demand parameter A increases? Prove your claim. [**10 marks**]

6. An economy comprises of a consolidated household sector, a firm sector and the government. The household supplies labour (L) to the firm.The firm produces a single good (Y) by means of a production function$Y = F(L); F' > 0, F'' < 0$, and maximizes profits $\Pi = PY - WL$,where P is the price of Y and W is the wage rate.The household, besides earning wages, is also entitled to the profits of the firm. The household maximizes utility (U), given by:

$$U = \frac{1}{2}\ln C + \frac{1}{2}\ln\frac{M}{P} - d(L);$$

where C is consumption of the good and $\frac{M}{P}$ is real balance holding. The term $d(L)$ denotes the disutility from supplying labour; with $d' > 0, d'' > 0$.The household's budget constraint is given by:

$$PC + M = WL + \Pi + \overline{M} - PT;$$

where $\overline{M}$ is the money holding the household begins with, M is the holding they end up with and T is the real taxes levied by the government. The government's demand for the good is given by G.The government's budget constraint is given by:

$$M - \overline{M} = PG - PT.$$

Goods market clearing implies: $Y = C + G$.
   (a) Prove that $\frac{dY}{dG} \in (0,1)$, and that government expenditure crowds out private consumption (i.e., $\frac{dC}{dG} < 0$). [**15 marks**]
   (b) Show that everything else remaining the same, a rise in $\overline{M}$ leads to an equi-proportionate rise in P. [**5 marks**]

7. Consider the Solow growth model in continuous time, where the exogenous rate of technological progress, g, is zero. Consider an intensive form production function given by:

$$f(k) = k^4 - 6k^3 + 11k^2 - 6k; \tag{1}$$

where $k = \frac{K}{L}$ (the capital labour ratio).

   (a) Specify the assumptions made with regard to the underlying extensive form production function $F(K, L)$ in the Solow growth model, and explain which ones among these assumptions are violated by (1). [**10 marks**]

   (b) Graphically show that, with a suitable value of $(n + \delta)$, where n is the population growth rate, and $\delta \in [0,1]$ is the depreciation rate on capital, there exist three steady state equilibria. [**5 marks**]

   (c) Explain which of these steady state equilibria are locally unstable, and which are locally stable. Also explain whether any of these equilibria can be globally stable. [**5 marks**]

8. Consider a standard Solow model in discrete time, with the law of motion of capital is given by

$$K(t+1) = (1-\delta)K(t) + I(t),$$

where $I(t)$ is investment at time *t* and$K(t)$ is the capital stock at time *t*; the capital stock depreciates at the rate $\delta \in [0,1]$.Suppose output, $Y(t)$, is augmented by government spending, $G(t)$, in every period, and that the economy is closed; thus:

$$Y(t) = C(t) + I(t) + G(t),$$

where $C(t)$ is consumption at time *t*. Imagine that government spending is given by:

$$G(t) = \sigma Y(t),$$

where $\sigma \in [0,1]$.

   (a) Suppose that: $C(t) = (\emptyset - \lambda\sigma)Y(t)$; where $\lambda \in [0,1]$.Derive the effect of higher government spending (in the form of higher σ) on the steady state equilibrium. [**10 marks**]

   (b) Does a higher σ lead to a lower value of the capital stock in every period (i.e, along the entire transition path)? Prove your claim. [**10 marks**]