Test code: ME I/ME II, 2004
Syllabus for ME I

Matrix Algebra: Matrices and Vectors, Matrix Operations, Determinants,

Nonsingularity, Inversion, Cramer's rule.

Calculus: Limits, Continuity, Differentiation of functions of one or more variables, Product rule, Partial and total derivatives,Derivatives of implicit functions, Unconstrained optimization (first and second order conditions for extrema of a single variable and several variables). Taylor Series, Definite and Indefinite Integrals: standard formulae, integration by parts and integration by substitution. Differential equations. Constrained optimization of functions of a single variable.

Theory of Sequence and Series:

Linear Programming: Formulations, statements of Primal and Dual problems. Graphical Solutions.

Theory of Polynomial Equations (up to third degree)

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory.

Sample Questions for ME I (Mathematics), 2004

For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question and write it in your answer book.

1. $X \sim B(n, p)$. The maximum value of Var (X) is

   (A) $\frac{n}{4}$; (B) n ; (C) $\frac{n}{2}$; (D) $\frac{1}{n}$.

2. P (x) is a quadratic polynomial such that P (1) = - P (2). If one root of the equation is -1, the other root is

   (A) $-\frac{4}{5}$; (B) $\frac{8}{5}$; (C) $\frac{4}{5}$; (D) $-\frac{8}{5}$.

3. $f(x) = (x-a)^3 + (x-b)^3 + (x-c)^3$, $a < b < c$.

   The number of real roots of $f(x) = 0$ is

   (A) 3; (B) 2; (C) 1; (D) 0.

4. A problem of statistics is given to the three students A, B and C. Their probabilities of solving it independently are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$, respectively.

   The probability that the problem will be solved is

   (A) $\frac{3}{5}$; (B) $\frac{2}{5}$; (C) $\frac{1}{5}$; (D) $\frac{4}{5}$.

5. Suppose correlation coefficients between x and y are computed from

   (i) $y = 2 + 3x$ and (ii) $2y = 5 + 8x$. Call them ρ_1 and ρ_2, respectively.

   Then

(A) $\rho_1 > \rho_2$; (B) $\rho_2 > \rho_1$; (C) $\rho_1 = \rho_2$; (D) either $\rho_1 > \rho_2$ or $\rho_1 < \rho_2$.

6. In the linear regression of y on x, the estimate of the slope parameter is given by $\frac{Cov(x,y)}{V(x)}$. Then the slope parameter for the linear regression of x on y is given by

(A) $\frac{V(x)}{Cov(x,y)}$; (B) $\frac{Cov(x,y)}{V(x)}$; (C) $\frac{Cov(x,y)}{\sqrt{V(x)V(y)}}$; (D) none of these.

7. Suppose $f(x) = e^x$ then

(A) $f\left(\frac{x_1 + x_2}{2}\right) > \frac{f(x_1) + f(x_2)}{2}$ for all x_1 and x_2 and $x_1 \neq x_2$;

(B) $f\left(\frac{x_1 + x_2}{2}\right) < \frac{f(x_1) + f(x_2)}{2}$ for all x_1 and x_2 and $x_1 \neq x_2$;

(C) $f\left(\frac{x_1 + x_2}{2}\right) > \frac{f(x_1) + f(x_2)}{2}$ for some values of x_1 and x_2 and $x_1 \neq x_2$;

$f\left(\frac{x_1 + x_2}{2}\right) < \frac{f(x_1) + f(x_2)}{2}$ for some values of x_1 and x_2 and $x_1 \neq x_2$;

(D) there exists at least one pair (x_1, x_2), $x_1 \neq x_2$ such that

$$f\left(\frac{x_1 + x_2}{2}\right) = \frac{f(x_1) + f(x_2)}{2}$$

8. Consider the series (i) and (ii) defined below:

(i) $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \ldots$

and

(ii) $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots$

Then,

(A) the first series converges, but the second series does not converge;

(B) the second series converges, but the first series does not converge;

(C) both converge;

(D) both diverge.

9. The function $x|x|$ is

(A) discontinuous at $x = 0$;

(B) continuous but not differentiable at $x = 0$;

(C) differentiable at $x = 0$;

(D) continuous everywhere but not differentiable anywhere.

10. The sequence $(-1)^{n+1}$ has

(A) no limit;

(B) 1 as the limit;

(C) –1 as the limit;

(D) 1 and –1 as the limits.

11. The series

$1.\frac{1}{2}+\frac{1}{2}.\frac{1}{3}+\frac{1}{3}.\frac{1}{4}+\ldots+\frac{1}{n}.\frac{1}{n+1}+\ldots$

(A) diverges;

(B) converges to a number between 0 and 1;

(C) converges to a number greater than 2;

(D) none of these.

12. Consider the function

$$f(x)=\frac{x^t-1}{x^t+1}, \qquad (x>0)$$

The limit of the function as t tends to infinity;

(A) does not exist; (B) exists and is everywhere continuous;

(C) exists and is discontinuous at exactly one point;

(D) exists and is discontinuous at exactly two points.

13. If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial x}$ is equal to

(A) *sin u cos u* ; (B) *cot u* ; (C) *tan u* ; (D) none of these.

14. The deviative of $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ with respect to x is

(A) $-\frac{x}{2}$; (B) $\frac{x}{2}$; (C) $\frac{1}{2}$; (D) $-\frac{1}{2}$.

15. The sum of the infinite series $1+\frac{3}{4}+\frac{7}{16}+\frac{15}{64}+\frac{31}{256}+\ldots$ is

(A) $\frac{2}{3}$; (B) $\frac{4}{3}$; (C) $\frac{8}{3}$; (D) none of these.

16. Five boys and four girls are to be seated in a row for a photograph. It is desired that no two girls sit together. The number of ways in which they can be so arranged is

(A) $6_{P_4}\times \lfloor 5$; (B) $4_{P_2}\times \lfloor 5$; (C) $\lfloor 4\times \lfloor 5$; (D) none of these.

17. A point moves so that the ratio of its distance from the points *(-a, 0)* and *(a, 0)* is 2:3. The equation of its locus is

(A) $x^2+y^2+10ax+a^2=0$;

(B) $5x^2+5y^2+26ax+5a^2=0$;

(C) $5x^2+5y^2-26ax+5a^2=0$;

(D) $x^2 + y^2 - 10ax + a^2 = 0$;

18. If the sum, $\sum_{x=1}^{100} \lfloor x$, is divided by 36, the remainder is

(A) 3; (B) 6; (C) 9; (D) none of these.

19. If a, b, c, d are in G.P., then $\left(a^3+b^3\right)^{-1}, \left(b^3+c^3\right)^{-1}, \left(c^3+d^3\right)^{-1}$ are in

(A) A. P.; (B) G. P.; (C) H.P.; (D) none of these.

20. The solution set of the inequality $\left| \, |x| - 1 \right| < 1 - x$ is

(A) $(-\infty, 0)$; (B) $(-\infty, \infty)$; (C) $(0, \infty)$; (D) $(-1, 1)$.

21. The distance of the curve, $y = x^2$, from the straight line $2x - y = 4$ is minimum at the point

(A) $(-1, 1)$; (B) $(1, 1)$; (C) $(2, 4)$; (D) $\left(\frac{1}{2}, \frac{1}{4}\right)$

22. The dual to the following linear program:

$$\begin{aligned} \text{maximise} \quad & x_1 + x_2 \\ \text{subject to} \quad & -3x_1 + 2x_2 \le -1 \\ & x_1 - x_2 \le 2 \\ & x_1 \ge 0, x_2 \ge 0 \end{aligned}$$

has

(A) a unique optimal solution;

(B) a feasible solution, but no optimal solution;

(C) multiple optimal solutions;

(D) no feasible solution.

23. The number of real roots of the equation

$x^2 - 3\,|x| + 2 = 0$

is

(A) 1; (B) 2; (C) 3; (D) 4.

24. There are four letters and four directed envelopes. The number of ways in which the letters can be put into the envelopes so that every letter is in a wrong envelope is

(A) 9; (B) 12; (C) 16; (D) 64.

25. If $a^2x^2 + 2bx + c = 0$ has one root greater than unity and the other less than unity, then

(A) $a^2 + 2b + c = 0$; (B) $a^2 + 2b + c > 0$;

(C) $2b + c < 0$; (D) $2b + c > 0$;

26. Given the two sequences $a_n = \frac{1}{n}$ and

$b_n = \frac{1}{n+1}$, the sum, $\sum_{n=1}^{99} \frac{(a_n - b_n)^2}{a_n b_n}$, is

(A) 1; (B) $1 - \frac{1}{99}$; (C) $\frac{99}{100}$; (D) none of these.

27. If $A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$, then $A^{100} + A^5$ is

(A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$; (B) $\begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$; (C) $\begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix}$; (D) none of these.

28. $\lim\limits_{x \to 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$ is

(A) 0; (B) 1; (C) $e^{-\frac{1}{2}}$; (D) $e^{-\frac{1}{6}}$.

29. The system of equations

$$x + y + z = 6$$

$$x + ay - z = 1$$

$$2x + 2y + bz = 12$$

has a unique solution if and only if

(A) $a \neq 1$; (B) $b \neq 2$; (C) $ab \neq 2a + b + 2$; (D) none of these.

30. The number of times $y = x^3 - 3x + 3$ intersects the x – axis is

(A) 0; (B) 1; (C) 2; (D) 3.

Syllabus for ME II (Economics)

Microeconomics: Theory of consumer behaviour, Theory of producer behaviour, Market forms and Welfare economics.

Macroeconomics: National income accounting, Simple model of income determination and Multiplier, IS – LM model, Aggregate demand and aggregate supply model, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2004

$\boxed{\text{NO. 1.}}$ ***Instruction for question numbers 1 (i) – 1 (vi)***

For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question and write it in you answer book.

(i) A consumer consumes only two goods x and y. Her utility function is $U(x, y) = x + y$. Her budget constraint is $px + y = 10$ where p is the price of good x. If $p = \frac{1}{2}$, then the (absolute) own – price elasticity of good x is

(A) 0; (B) $\frac{1}{2}$; (C) 1; (D) ∞

(ii) A consumer consumes only two goods x and y. The price of good x in the local market is p and that in a distant market is q, where $p > q$. However, to go to the distant market, the consumer has to incur a fixed cost C. Suppose that the price of good y is unity in both markets. The consumer's income is I and $I > C$. Let x_0 be the equilibrium consumption of good x. If the consumer has smooth downward sloping and convex indiference curves, then

(A) $(p-q)x_0 = C$ always holds;

(B) $(p-q)x_0 = C$ never holds;

(C) $(p-q)x_0 = C$ may or may not hold depending on the consumer's preferences;

(D) none of the above.

(iii) Consider the following production function $Q = \min\left(\frac{L}{2a}, \frac{K}{4b}\right)$.

Let w and r be the wage and rental rate respectively. The cost function associated with this production function is

(A) 2 awQ;

(B) 4 brQ;

(C) $(wa + 2br)Q$;

(D) none of the above.

(iv) During a period net loan from abroad of an economy is positive. This necessarily implies that during this period

(A) trade balance is positive;

(B) net factor income from abroad is negative;

(C) current account surplus is negative;

(D) change in foreign exchange reserve is positive.

(v) Consider a simple Keynesian economy in which the government expenditure (G) exactly equals its total tax revenue: $G = tY$ where t is the tax rate and Y is the national income. Suppose that the government raises t. Then

(A) Y increases;

(B) Y decreases;

(C) Y remains unchanged;

(D) Y may increase or decrease.

(vi) Which one of the following statements is FALSE?

Interest on pubic debt is not a part of

(A) both personal income and national income;

(B) government consumption expenditure;

(C) national income;

(D) personal income.

No. 2 Indicate whether the following statements are TRUE or FALSE, adding a few lines to justify your answer in each case:

(i) A barrel of crude oil yields a fixed number of gallons of gasoline. Therefore, the price per gallon of gasoline divided by the price per barrel of crude oil is independent of crude oil production.

(ii) If there is no money illusion, once you know all the price elasticities of demand for a commodity, you can calculate its income elasticity.

(iii) If two agents for an Edgeworth box diagram have homothetic preferences then the contract curve is a straight line joining the two origins.

No. 3 Consider a duopoly situation where the inverse market demand function is P(Q) = 10- Q $\left(\text{where } Q = q_1 + q_2\right)$. The cost function of firm 1 is $\left(1 + 2q_1\right)$ and that of firm 2 is $\left(1 + 4q_2\right)$. The firms do not incur any fixed cost if they produce nothing. Calculate the Cournot equilibrium output and profit of the two firms. If, ceteris paribus, the fixed cost of firm 2 is Rs. 2 (instead of Re. 1), what happens to the Cournot equilibrium?

$\boxed{\text{No. 4}}$ A simple Keynesian model has two groups of income earners. The income of group 1 (Y_1) is fixed at Rs. 800. Both groups have proportional consumption function; the average propensity to consume is 0.8 for group 1 and 0.5 for group 2. Group 2 consumes only domestically produced goods. However, group 1 consumes both domestically produced as well as imported goods, their marginal propensity to import being 0.4. investment goods are produced domestically and Investment (I) is autonomously given at Rs. 600.

(i) Compute gross domestic product (Y)

(ii) Suppose group 2 makes an income transfer of Rs. 100 to group 1. However, imports are restricted and cannot exceed Rs. 250 (that is, import function ceases to be operative at this value). How does Y change?

(iii) How does your answer to part (ii) change, if the upper limit of imports is raised to Rs. 400?

$\boxed{\text{No. 5}}$ A consumer consumes electricity (X_E) and other goods (X_O). The price of other goods is unity. To consume electricity the consumer has to pay a rental charge *R* and a per unit price *p*. However, *p* increases with the quantity of electricity consumed according to the function $p = \frac{1}{2} X_E$.

The utility function of the consumer is $U = X_E + X_O$ and his income is $I > R$.

(i) Draw the budget line of the consumer.

(ii) If $R = 0$ and $I = 1$, find the optimum consumption bundle.

(iii) Find the maximum R that the electricity company can extract from the consumer.

$\boxed{\text{No. 6}}$ A consumer has Rs. 25 to spend on two goods x and y. The price of good x is Rs. 3 and that of good y is Rs. 4. The continuously differentiable utility function of the consumer is $U(x, y) = 12x + 16y - x^2 - y^2$ where $x \geq 0$ and $y \geq 0$. What happens to the optimum commodity bundle if, instead of Rs. 25, the consumer has Rs. 50 or more to spend on the two goods?

$\boxed{\text{No. 7}}$ Consider an IS – LM model with the following elements:

$$s = s\left(y - t.\theta.y\right), \qquad 0 < s'(.) < 1 \qquad (1)$$

$$i = i\,(r), \quad i'(.) < 0 \qquad (2)$$

$$l = l\,(y, r), \quad l_1(.) > 0, \quad l_2(.) < 0 \qquad (3)$$

where s is the desired private saving, y is the real GNP, θ is the (exogenously given) labour's share in GNP, t is the proportionate tax rate on labour earnings, i is the desired real physical investment, r is the interest rate and l is the desired real money holdings. The real money balance $\frac{M}{P}$, together with the real government spending g and the tax – rate t, are exogenous.

(i) The Laffer curve plots equilibrium tax collections on the vertical axis against t on the horizontal axis. Laffer's famous formula was that the curve slopes downwards. Show analytically whether or not that can happen here.

(ii) Suppose t is imposed on all factor payments, that is, on GNP. Reformulate equation (1) for this case. Does your answer to part (i) change?