

Option A (Series 02)

June 26, 2010

Time : 3 Hours

Maximum Marks : 100

General Instructions. Please read the following instructions carefully :

- Check that you have a bubble-sheet accompanying this examination booklet. Do **not** break the seal on this booklet until instructed to do so by the invigilator.
 - Immediately on receipt of this booklet, fill in your Signature, Name, Roll number and Answer sheet number (**see the top left-hand-side of the bubble-sheet**) in the space provided below.
 - This examination will be checked by a machine. **Therefore, it is very important that you follow the instructions on the bubble-sheet.**
 - Fill in the required information in Boxes 1, 2, 4,5 and 6 on the bubble-sheet. The invigilator will sign in Box 3.
 - Do **not** disturb your neighbours at any time.
 - Make sure you do not have calculators, mobile telephones, papers, books, etc., on your person. **Anyone engaging in illegal examination practices will be immediately evicted and that person's candidature will be cancelled.**
 - When you finish the examination, hand in this booklet **and** the bubble-sheet to the invigilator.
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Signature

Name

Roll number

Answer sheet number

Part I

Instructions.

- Check that this booklet has pages 1 through 22. Also check that the bottom of each page is marked with *EEE 2010 A 02*.
- This part of the examination consists of 20 multiple-choice questions. Each question is followed by four possible answers, at least one of which is correct. If more than one choice is correct, choose **only the best one**. Among the correct answers, the best answer is the one that implies (or includes) the other correct answer(s). Indicate your chosen best answer on the **bubble-sheet** by shading the appropriate bubble.
- For each question, you will get 1 mark if you choose only the best answer. If you choose none of the answers, then you will get 0 for that question. **However, if you choose something other than the best answer or multiple answers, then you will get $-\frac{1}{3}$ mark for that question.**
- You may use the blank pages at the end of this booklet, marked **Rough work**, to do your calculations and drawings. No other paper will be provided for this purpose. Your "Rough work" will not be read or checked.

You may begin now. Good luck!

QUESTION 1. A number, say X_1 , is chosen at random from the set $\{1, 2\}$. Then a number, say X_2 , is chosen at random from the set $\{1, X_1\}$. The probability that $X_1 = 2$ given that $X_2 = 1$ is

- (a) 1
- (b) $\frac{1}{2}$
- (c) $\frac{1}{3}$
- (d) $\frac{1}{4}$

QUESTION 2. Suppose a random variable X takes values $-2, 0, 1$ and 4 with probabilities $0.4, 0.1, 0.3$ and 0.2 respectively.

- (a) The unique median of the distribution is 1
- (b) The unique median of the distribution is 0
- (c) The unique median of the distribution lies between 0 and 1
- (d) The distribution has multiple medians

QUESTION 3. Two persons, A and B , shoot at a target. Suppose the probability that A will hit the target on any shot is $\frac{1}{3}$ and the probability that B will hit the target on any

shot is $1/4$. Suppose A shoots first and they take turns shooting. What is the probability that the target is hit for the first time by A 's third shot?

- (a) $1/24$
- (b) $1/12$
- (c) $1/6$
- (d) $1/3$

QUESTION 4. A fair coin is tossed repeatedly until a head is obtained for the first time. Let X denote the number of tosses that are required. The value of the distribution function of X at 3 is

- (a) $3/4$
- (b) $1/2$
- (c) $7/8$
- (d) $7/16$

QUESTION 5. Using a random sample, an ordinary least squares regression of Y on X yields the 95% confidence interval $0.43 < \beta < 0.59$ for the slope parameter β . Which of the following statements is false?

- (a) This interval contains the parameter β with probability 0.95
- (b) The point estimate of β obtained from our regression always lies within this interval
- (c) The 90% confidence interval for β is a subset of the interval obtained above
- (d) If such intervals are constructed from repeated samples drawn from the population in question, then on average 95 out of 100 of these intervals are likely to contain the true parameter value.

QUESTION 6. Consider a binary relation \succeq defined on the set $A = \{x, y, z\}$. Define relations \succ and \sim on A by: for $a, b \in A$,

$$a \succ b \text{ if and only if } a \succeq b \text{ and not } b \succeq a$$

and

$$a \sim b \text{ if and only if } a \succeq b \text{ and } b \succeq a$$

Suppose $x \succ y$, $y \sim z$ and $x \sim z$. Then

- (a) \succ is transitive
- (b) \sim is transitive
- (c) both \succ and \sim are transitive
- (d) we cannot conclude anything about the transitivity of \succ and \sim

QUESTION 7. The utility function $u(x, y) = (x + y)^{1/2}$, for $(x, y) \geq (0, 0)$, exhibits

- (a) diminishing marginal rate of substitution and diminishing marginal utilities
- (b) increasing marginal rate of substitution and diminishing marginal utilities
- (c) constant marginal rate of substitution and diminishing marginal utilities
- (d) increasing marginal rate of substitution and constant marginal utilities

QUESTION 8. Suppose there are just two goods, say x_1 and x_2 . Consider a consumer who chooses $x_2 = 0$ for all income levels $w > 0$ and all prices $p_1 > 0$ and $p_2 > 0$. These choices are consistent with the consumer

- (a) having utility function $u(x_1, x_2) = x_1 + 2x_2$
- (b) having utility function $u(x_1, x_2) = 2x_1 + x_2$
- (c) lexicographically preferring x_2 to x_1
- (d) lexicographically preferring x_1 to x_2

QUESTION 9. Consider a Cournot duopoly with firms 1 and 2 that produce a homogeneous good. The inverse demand curve for this good is given by $P(x) = 5 - x$, where x is the total output of the two firms. Firm 1 has a constant average cost $5/2$ and firm 2 has a constant average cost $3/2$. In equilibrium,

- (a) only firm 1 produces a positive output
- (b) only firm 2 produces a positive output
- (c) both firms produce positive outputs with firm 1 producing more than firm 2
- (d) both firms produce positive outputs with firm 2 producing more than firm 1

QUESTION 10. If all input prices double, then what happens to the minimum cost of producing a given output?

- (a) It doubles
- (b) It more than doubles
- (c) It less than doubles
- (d) It depends on the production function

QUESTION 11. In models where expectations adjust slowly, a progressive income tax schedule is

- (a) a means to maximize tax revenue
- (b) a means to ensure tax compliance
- (c) always an automatic stabilizer in the long run
- (d) often an automatic stabilizer in the short run

QUESTION 12. Open market operations decrease the supply of base money by

- (a) selling government bonds
- (b) selling gold

- (c) reducing foreign currency holdings
- (d) all of the above

QUESTION 13. If an economy is experiencing hyperinflation, then

- (a) government seigniorage goes up
- (b) government seigniorage goes down
- (c) government seigniorage remains unchanged
- (d) the impact on government seigniorage is ambiguous

QUESTION 14. When the nominal interest rate changes but the real rate of interest remains unchanged

- (a) it affects both the investment function as well as the money demand function
- (b) it affects the investment function but does not affect the money demand function
- (c) it affects the money demand function but does not affect the investment function
- (d) neither the investment function nor the money demand function gets affected

QUESTION 15. *Ceteris paribus*, higher velocity of money circulation leads to

- (a) an increase in both the real and the nominal demand for money
- (b) an increase in the real demand for money and a decrease in the nominal demand for money
- (c) a decrease in the real demand for money and an increase in the nominal demand for money
- (d) a decrease in both the real and the nominal demand for money

QUESTION 16. Consider a 3×3 nonsingular matrix A with real entries. If the matrix B is derived from A by interchanging the first and last columns of A , then the determinant of B , denoted $\det B$, is equal to

- (a) $\det A$
- (b) $-\det A$
- (c) 0
- (d) $1/\det A$

QUESTION 17. The sequence $(-1)^n(1 + 1/n)$, for positive integers n ,

- (a) has limit point 1
- (b) has limit point -1
- (c) has limit points 1 and -1
- (d) has no limit points

QUESTION 18. Consider the following two games:

		Hawk			Hawk	Dove				
Game 1 :	Enter	(-1, 1)	Game 2 :	Enter	(-1, 1	3, 3)
	Not enter		0, 6			Not enter		0, 6	0, 7	

In every payoff pair " x, y ", x is the payoff of the row-player and y is the payoff of the column-player. Analyze these games. These games illustrate that, in a strategic situation

- (a) an expanded set of strategic options can be disadvantageous
- (b) a contracted set of strategic options can be advantageous
- (c) one should view the situation from one's own, as well as from one's opponent's, perspective
- (d) all of the above are true

QUESTION 19. Ice-cream vendors A and B know that they have to locate simultaneously on a beach. The beach is identified with the interval $[0, 1]$ and at every point in $[0, 1]$ there is a person who wants exactly one ice-cream cone. Each person will buy the ice-cream from the nearest vendor; if there are equidistant vendors, then the buyer randomizes among them with equal probabilities. Each vendor wants to maximize his own expected market share. The vendors will locate at

- (a) 0 and 1
- (b) $1/4$ and $3/4$
- (c) $1/2$ and $1/2$
- (d) $1/3$ and $2/3$

QUESTION 20. Suppose there is only one future period and the (presently unknown) state of the world in that period can be either s_1 or s_2 . The future return on a share of a given company is 5 in state s_1 and -1 in state s_2 . The future return on a government bond is 1 independent of the state. Suppose a third asset is offered on the market whose return is 3 in state s_1 and 0 in state s_2 . The current prices of the stock and the bond are 3 and 1 respectively. If the price of the new asset rules out the possibility of arbitrage profit (which arises when portfolios of assets that are identical in terms of returns have different prices), what is the price of the new asset?

- (a) It depend on the probabilities of the future states
- (b) Strictly between 2 and 3
- (c) Strictly between 1 and 2
- (d) 2

End of Part I.

Proceed to Part II of the examination on the next page.

Part II

Instructions.

• This part of the examination consists of 40 multiple-choice questions. Each question is followed by four possible answers, at least one of which is correct. If more than one choice is correct, choose **only the best one**. Among the correct answers, the best answer is the one that implies (or includes) the other correct answer(s). Indicate your chosen best answer on the **bubble-sheet** by shading the appropriate bubble.

• For each question, you will get 2 marks if you choose only the best answer. If you choose none of the answers, then you will get 0 for that question. **However, if you choose something other than the best answer or multiple answers, then you will get $-2/3$ mark for that question.**

The following notational conventions apply wherever the following symbols are used. \mathfrak{R} denotes the set of real numbers. Given a function f , $Df(x)$ and $D^2f(x)$ denote the first and second derivatives of f (if they exist), respectively, evaluated at x .

QUESTION 21. Suppose X_1 and X_2 are real-valued random variables with f as their common probability density function. Suppose (x_1, x_2) is a sample generated by these random variables. The expectation of the number of observations in the sample that fall within a specified interval $[a, b]$ is

- (a) $\left(\int_a^b f(x) dx\right)^2$
- (b) $\int_a^b x^2 f(x) dx$
- (c) $2 \int_a^b f(x) dx$
- (d) $\int_a^b xf(x) dx$

QUESTION 22. Suppose X_1, \dots, X_n are observed completion times of an experiment with values in $[0, 1]$. Each of these random variables is uniformly distributed on $[0, 1]$. If Y is the maximum observed completion time, then the mean of Y is

- (a) $[n/(n+1)]^2$
- (b) $n/2(n+1)$
- (c) $n/(n+1)$
- (d) $2n/(n+1)$

QUESTION 23. Suppose the random variable X takes values in the set $\{-1, 0, 1\}$ and the probability of each value is equal. Let $Y = X^2$. Which of the following statements is true?

- (a) X and Y are correlated but independent
- (b) X and Y are uncorrelated but dependent
- (c) X and Y are dependent and have the same mean
- (d) X and Y are correlated and have different means

QUESTION 24. Suppose player 1 has five coins and player 2 has four coins. Both players toss all their coins and observe the number that come up heads. Assuming all the coins are fair, what is the probability that player 1 obtains more heads than player 2?

- (a) $1/2$
- (b) $4/9$
- (c) $5/9$
- (d) $4/5$

QUESTION 25. Suppose 10 athletes are running in a race and exactly 2 of them are taking banned drugs. An investigator randomly selects 2 athletes for drug-testing. What is the probability that neither of the cheaters will be caught?

- (a) $16/25$
- (b) $4/5$
- (c) $3/5$
- (d) $28/45$

QUESTION 26. Suppose θ is a random variable with uniform distribution on the interval $[-\pi/2, \pi/2]$. The value of the distribution function of the random variable $X = \sin \theta$ at $x \in [-1, 1]$ is

- (a) $\sin^{-1}(x)$
- (b) $\sin^{-1}(x) + \pi/2$
- (c) $\sin^{-1}(x)/\pi + 1/2$
- (d) $\sin^{-1}(x)/\pi + \pi/2$

QUESTION 27. Let X be a normally distributed random variable with mean 0 and variance σ^2 . Then, the mean of X^2 is

- (a) 0
- (b) σ
- (c) 2σ
- (d) σ^2

The next three questions are based on the following data. The number of loaves of bread sold by a bakery in a day is a random variable X . The distribution of X has a

probability density function f given by

$$f(x) = \begin{cases} kx, & \text{if } x \in [0, 5) \\ k(10 - x), & \text{if } x \in [5, 10) \\ 0, & \text{if } x \in [10, \infty) \end{cases}$$

QUESTION 28. As f is a probability density function, the value of k must be

- (a) 0
- (b) $-2/25$
- (c) $1/25$
- (d) $2/75$

QUESTION 29. Let A be the event that $X \geq 5$ and let B be the event that $X \in [3, 8]$. The probability of A conditional on B is

- (a) $16/37$
- (b) $21/37$
- (c) $25/37$
- (d) 1

QUESTION 30. Events A and B are

- (a) not independent
- (b) independent
- (c) conditionally independent
- (d) unconditionally independent

The next four questions are based on the following data. Consider an exchange economy with agents 1 and 2 and goods x and y . Agent 1's endowment is $(0, 1)$ (i.e., no good x and 1 unit of good y) and agent 2's endowment is $(2, 0)$ (i.e., 2 units of good x and no good y). The agents can consume only nonnegative amounts of x and y .

QUESTION 31. Suppose agent 1 lexicographically prefers x to y , i.e., between any two bundles of goods, she strictly prefers the bundle containing more of x , and if the bundles contain equal amounts of x , then she strictly prefers the bundle with more of y . Suppose agent 2 treats x and y as perfect substitutes, i.e., between any two bundles (x, y) and (x', y') , she strictly prefers (x, y) if and only if $x + y > x' + y'$.

The competitive equilibrium allocation for this economy is

- (a) 1 gets $(0, 1)$ and 2 gets $(2, 0)$
- (b) 1 gets $(2, 0)$ and 2 gets $(0, 1)$
- (c) 1 gets $(3/2, 0)$ and 2 gets $(1/2, 1)$

- (d) 1 gets (1,0) and 2 gets (1,1)

QUESTION 32. Suppose agents 1 and 2 have the preferences described above. The set of all possible competitive equilibrium prices consists of all $p_x > 0$ and $p_y > 0$ such that

- (a) $p_x/p_y = 1$
- (b) $p_x/p_y \geq 1$
- (c) $p_x/p_y \leq 1$
- (d) $p_x/p_y > 0$

QUESTION 33. Now suppose agent 1 lexicographically prefers y to x and agent 2 treats x and y as perfect substitutes. The set of all possible competitive equilibrium prices consists of all $p_x > 0$ and $p_y > 0$ such that

- (a) $p_x/p_y = 1$
- (b) $p_x/p_y \geq 1$
- (c) $p_x/p_y \leq 1$
- (d) $p_x/p_y > 0$

QUESTION 34. Now suppose agent 1 lexicographically prefers y to x and agent 2 treats x and y as perfect complements. The set of competitive equilibrium allocations

- (a) includes the allocation (1, 0) for agent 1 and (1, 1) for agent 2
- (b) includes the allocation (0, 1) for agent 1 and (2, 0) for agent 2
- (c) is empty
- (d) includes all allocations $(x, 1)$ for agent 1 and $(2 - x, 0)$ for agent 2, where $x \in [0, 2]$

QUESTION 35. Consider a person who chooses among lotteries. Each lottery is of the form (p_1, p_2, p_3) , where p_1 is the probability of getting Rs. 5, p_2 is the probability of getting Rs. 1 and p_3 is the probability of getting Rs. 0. This person prefers lottery (0, 1, 0) to lottery (0.1, 0.89, 0.01). If this person maximizes expected utility and is faced with the lotteries (0, 0.11, 0.89) and (0.1, 0, 0.9), which lottery should he prefer?

- (a) The lottery (0, 0.11, 0.89)
- (b) The lottery (0.1, 0, 0.9)
- (c) He should be indifferent between these lotteries
- (d) There is insufficient data to decide

QUESTION 36. Consider an economy with two agents, A and B , and two goods, x_1 and x_2 . Both agents treat x_1 and x_2 as perfect complements. Suppose the total endowment of x_1 is 4 and the total endowment of x_2 is 2. Which of the following allocations is not Pareto optimal? (Note that a bundle (a, b) represents a units of x_1 and b units of x_2 .)

- (a) A gets (1, 1) and B gets (1, 1)

- (b) A gets $(2, 1)$ and B gets $(3/2, 1)$
- (c) A gets $(1/2, 3/2)$ and B gets $(3, 1/2)$
- (d) A gets $(3, 2)$ and B gets $(0, 0)$

QUESTION 37. A consumer has the utility function $u(x, y) = xy$. Suppose the consumer demands bundle (x^*, y^*) . Now suppose the seller of good x offers a “buy one, get one free” scheme: for each unit of good x purchased, the consumer gets another unit of x for free. Given this scheme, suppose the consumer buys bundle (x_d, y_d) and gets an additional x_d for free. Which one of the following statements must be true?

- (a) $x_d > x^*$ and $y_d > y^*$
- (b) $x_d > x^*$ and $y_d = y^*$
- (c) $x_d > x^*$ and $y_d < y^*$
- (d) $x_d = x^*$ and $y_d = y^*$

QUESTION 38. Consider a Bertrand duopoly with firms 1 and 2 that produce a homogeneous good and set prices p_1 and p_2 respectively. Suppose p_1 and p_2 have to be positive integers. If $p_1 < p_2$ (resp. $p_1 > p_2$), then firm 1 (resp. firm 2) sells $5 - p_1$ (resp. $5 - p_2$) and the other firm sells nothing. If $p_1 = p_2$, then each firm sells $(5 - p_1)/2$. Firm 1 has a constant average cost $5/2$ and firm 2 has a constant average cost $3/2$. In equilibrium

- (a) $p_1 = 2 = p_2$
- (b) $p_1 = 3 = p_2$
- (c) $p_1 = 3$ and $p_2 = 2$
- (d) $p_1 = 3$ and p_2 is 2 or 3

QUESTION 39. Consider a Stackelberg duopoly with firm 1 as the leader and firm 2 as the follower. If (q_1, q_2) is the Stackelberg equilibrium, then

- (a) firm 1’s optimal isoprofit curve and firm 2’s reaction curve intersect at (q_1, q_2) and are tangential at (q_1, q_2)
- (b) firm 2’s optimal isoprofit curve and firm 1’s reaction curve intersect at (q_1, q_2) and are tangential at (q_1, q_2)
- (c) isoprofit curves of the two firms intersect at (q_1, q_2) and are tangential at (q_1, q_2)
- (d) reaction curves of the two firms intersect at (q_1, q_2)

QUESTION 40. Firm 1 is the potential entrant into a market in which firm 2 is the incumbent monopolist. Firm 1 moves first and chooses to “enter” or “not enter”. If it does “not enter”, then firm 1 gets profit 0 and firm 2 gets the monopoly profit 10. If firm 1 “enters”, then firm 2 chooses to “fight” or “not fight”. If firm 2 fights, then firm 1’s profit is -2 and firm 2’s profit is 6. If firm 2 does “not fight”, then firm 1’s profit is 2 and firm 2’s profit is 8. Firm 2’s strategy of “fight” is best interpreted as

- (a) a commitment
- (b) a non-credible threat
- (c) a punitive action
- (d) acquiescence

QUESTION 41. Consider a closed economy. If the nominal wage is flexible and nominal money supply is increased, then which of the following will be true in equilibrium?

- (a) Real wage decreases and real money supply decreases
- (b) Real wage decreases and real money supply increases
- (c) Real wage is unchanged and real money supply is unchanged
- (d) Real wage decreases and real money supply is unchanged

QUESTION 42. Suppose an economy is at less than full employment and it consists of an aggregate “worker” and an aggregate “capitalist”, with the former having a higher marginal propensity to consume from his disposable income. Suppose both agents pay income tax according to the same linear schedule. If the government’s budget is in balance and a lump-sum income transfer is made from the capitalists to the workers, then the government’s

- (a) budget will go into deficit
- (b) budget will go into surplus
- (c) income and expenditure will be unchanged
- (d) income and expenditure will change but the budget will stay in balance

The next four questions are based on the following information. Consider an economy with an aggregate production function $Y = \alpha K + \beta L$, where α and β are positive constants, K is capital, L is labour and Y is output. K is fixed in the short run. Perfectly competitive producers take the nominal wage rate W and the price level P as given, and employ labour so as to maximize profit. This generates the labour demand schedule. The labour supply schedule is $L^S = -\gamma + \delta W/P$, where γ and δ are positive constants. Producers and workers have perfect information about P and W .

QUESTION 43. The labour market will clear if and only if

- (a) $\beta > \gamma/\delta$
- (b) $\beta < \gamma/\delta$
- (c) $\beta > \delta/\gamma$
- (d) $\beta < \delta/\gamma$

QUESTION 44. Assume that the required parametric condition of the previous question holds and that the nominal wage rate is fixed. The short run aggregate supply schedule

for this economy, with P along the vertical axis and Y along the horizontal axis, will look as follows:

(a) for high values of P it will be horizontal; for some mid-range values of P it will be downward sloping; for low values of P it will be horizontal again

(b) for high values of P it will be horizontal; for some mid-range values of P it will be upward sloping; for low values of P it will be horizontal again

(c) for high values of P it will be vertical; for some mid-range values of P it will be downward sloping; for low values of P it will be vertical again

(d) for high values of P it will be vertical; for some mid-range values of P it will be upward sloping; for low values of P it will be vertical again

QUESTION 45. If there is a one shot increase in the fixed stock of the capital stock, then the short run aggregate supply schedule will

(a) shift up

(b) shift down

(c) shift to the left

(d) shift to the right

QUESTION 46. If there is a one shot increase in the fixed nominal wage rate, then the short run aggregate supply schedule will

(a) shift up

(b) shift down

(c) shift to the left

(d) shift to the right

The next four questions are based on the following information. Consider a closed economy simple Keynesian model of the goods market, where prices are fixed and output in equilibrium is determined by aggregate demand. Investment is fixed at I^* . There is no government sector. Suppose there are two groups of households, called A and B, and the total income Y is distributed among these two groups in such a way that group A gets $Y^A = \lambda Y$, and group B gets $Y^B = (1 - \lambda)Y$, where $\lambda \in (0, 1)$ is a constant. The consumption function of group A is $C^A = c + c_A Y^A$ and the consumption function of group B is $C^B = c + c_B Y^B$, where $0 < c_A < c_B < 1$, i.e., the two groups have different consumption propensities.

QUESTION 47. The value of the investment multiplier in this economy is given by

(a) $\frac{1}{1 - \lambda c_A - \lambda c_B}$

(b) $\frac{1}{1 - (1 - \lambda)c_A - (1 - \lambda)c_B}$

- (c) $\frac{1}{1-c_B+\lambda(c_B-c_A)}$
 (d) $\frac{1}{1-c_A+(1-\lambda)(c_B-c_A)}$

QUESTION 48. If there is a one shot increase in the parameter λ ,

- (a) equilibrium output unambiguously increases
 (b) equilibrium output unambiguously decreases
 (c) equilibrium output remains unchanged
 (d) equilibrium increases or decreases depending on whether $\lambda \geq 1/2$

QUESTION 49. If there is a one shot increase in the parameter c_A ,

- (a) equilibrium output unambiguously increases
 (b) equilibrium output unambiguously decreases
 (c) equilibrium output remains unchanged
 (d) equilibrium increases or decreases depending on whether $\lambda \geq 1/2$

QUESTION 50. If there is a one shot increase in the parameter c_B ,

- (a) equilibrium output unambiguously increases
 (b) equilibrium output unambiguously decreases
 (c) equilibrium output remains unchanged
 (d) equilibrium increases or decreases depending on whether $\lambda \geq 1/2$

QUESTION 51. Suppose the function $f : \mathbb{R} \rightarrow \mathbb{R}$ is given by $f(x) = x^3 - 3x + b$. Find the number of points in the closed interval $[-1, 1]$ at which $f(x) = 0$.

- (a) None
 (b) At most one
 (c) One
 (d) At least one

QUESTION 52. Suppose a function $f : \mathbb{R} \rightarrow \mathbb{R}$ is differentiable at x . Consider the statements:

$$Df(x) = \lim_{h \rightarrow 0} \frac{f(x) - f(x-h)}{h} \quad (i)$$

and

$$Df(x) = \lim_{h \rightarrow 0} \frac{f(x+2h) - f(x+h)}{2h} \quad (ii)$$

In general,

- (a) (i) is true and (ii) is false
 (b) (i) is false and (ii) is true
 (c) Both are true
 (d) Both are false

QUESTION 53. Consider the statements: for $x, y \in \mathfrak{R}$,

$$|x| - |y| \leq |x - y| \quad (i)$$

and

$$||x| - |y|| = |x - y| \quad (ii)$$

In general,

- (a) (i) is true and (ii) is false
- (b) (i) is false and (ii) is true
- (c) Both (i) and (ii) are true
- (d) Both (i) and (ii) are false

QUESTION 54. Suppose the function $f : \mathfrak{R}^2 \rightarrow \mathfrak{R}$ is increasing in both arguments, i.e., $f(x, y)$ is increasing in x and increasing y . For $x, y \in \mathfrak{R}$, let

$$x \wedge y = \begin{cases} x, & \text{if } x \leq y \\ y, & \text{if } x > y \end{cases}$$

Define $g : \mathfrak{R}^2 \rightarrow \mathfrak{R}$ by

$$g(x, y) = \begin{cases} f(x, y) - \frac{1}{2}f(x \wedge y, x \wedge y), & \text{if } x \geq y \\ \frac{1}{2}f(x \wedge y, x \wedge y), & \text{if } x < y \end{cases}$$

Which of the following statements is correct?

- (a) g is increasing in x and decreasing in y
- (b) g is increasing in both x and y
- (c) g is increasing in x but may or may not be increasing in y
- (d) g may or may not be increasing in x

The next three questions are based on the following definitions. Consider the set $A = \{(x, y) \in \mathfrak{R}^2 \mid x^2 + y^2 \leq 1\}$. Given $(a, b) \in \mathfrak{R}^2$ such that $(a, b) \neq (0, 0)$ and $c > 1$, let

$$X = \{(x, y) + (a, b) \mid (x, y) \in A\}$$

$$Y = \{(2^{-1/2}(x + y), 2^{-1/2}(x - y)) \mid (x, y) \in A\}$$

$$Z = \{c(x, y) \mid (x, y) \in A\}$$

QUESTION 55. Which set is **not** a disc (i.e., the region inside a circle)?

- (a) X
- (b) Y

- (c) Z
- (d) None

QUESTION 56. Which set has a larger area than A ?

- (a) X
- (b) Y
- (c) Z
- (d) None

QUESTION 57. Which set does not contain all the points that belong to A ?

- (a) X
- (b) Y
- (c) Z
- (d) None

QUESTION 58. Consider a twice differentiable function $f : \mathcal{R} \rightarrow \mathcal{R}$ and $a, b \in \mathcal{R}$ such that $a < b$, $f(a) = 0 = f(b)$ and $D^2f(x) + Df(x) - 1 = 0$ for every $x \in [a, b]$. Then,

- (a) f has a maximum but not a minimum over the open interval (a, b)
- (b) f has a minimum but not a maximum over (a, b)
- (c) f has neither a maximum nor a minimum over (a, b)
- (d) f has a maximum and a minimum over (a, b)

QUESTION 59. Consider f as described in the previous question. Then,

- (a) $f(x) \leq 0$ for every $x \in [a, b]$
- (b) $f(x) \geq 0$ for every $x \in [a, b]$
- (c) $f(x) = 0$ for every $x \in [a, b]$
- (d) f must take positive and negative values on the interval $[a, b]$

QUESTION 60. Suppose $f : [0, 1] \rightarrow [0, 1]$ is a continuous nondecreasing function with $f(0) = 0$ and $f(1) = 1$. Define $g : [0, 1] \rightarrow [0, 1]$ by $g(y) = \min\{x \in [0, 1] \mid f(x) \geq y\}$. Then,

- (a) g is non-decreasing
- (b) If g is continuous, then f is strictly increasing
- (c) Neither (a) nor (b) is true
- (d) Both (a) and (b) are true