Delhi School of Economics
Department of Economics

Entrance Examination for M. A. Economics
Option B
June 29, 2013

Time 3 hours **Maximum marks** 100

Instructions Please read the following instructions carefully.

- Do **not** break the seal on this booklet until instructed to do so by the invigilator. Anyone breaking the seal prematurely will be evicted from the examination hall and his/her candidature will be cancelled.
- Fill in your Name and Roll Number on the detachable slip below.
- When you finish, hand in this examination booklet to the invigilator.
- Use of any electronic device (e.g., telephone, calculator) is **strictly prohibited** during this examination. Please leave these devices in your bag and away from your person.
- Do **not** disturb your neighbours for any reason at any time.
- **Anyone engaging in illegal examination practices will be immediately evicted and that person's candidature will be cancelled.**

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Marks tally

Question	Marks
I.1-10	
II.11	
II.12	
II.13	
II.14	
II.15	
Total	

Part I

Instructions.

• Check that this examination has pages 1 through 22.

• This part of the examination consists of 10 multiple-choice questions. Each question is followed by four possible answers, at least one of which is correct. If more than one choice is correct, choose **only the best one**. Among the correct answers, the best answer is the one that implies (or includes) the other correct answer(s). **Indicate your chosen answer by circling (a), (b), (c) or (d).**

• For each question, you will get 2 marks if you choose only the best answer. If you choose none of the answers, then you will get 0 for that question. **However, if you choose something other than the best answer or multiple answers, then you will get $-2/3$ mark for that question.**

You may begin now. Good luck!

QUESTION 1. Two women and four men are to be seated randomly around a circular table. Find the probability that the women are **not** seated next to each other.

(a) $1/2$
(b) $1/3$
(c) $2/5$
(d) $3/5$

QUESTION 2. A fair coin is tossed until a head comes up for the first time. The probability of this happening on an odd-numbered toss is

(a) $1/2$
(b) $1/3$
(c) $2/3$
(d) $3/4$

QUESTION 3. Let $f(x) = x + |x| + (x-1) + |x-1|$ for $x \in \Re$.

(a) f differentiable everywhere except at 0.
(b) f is not continuous at 0.
(c) f is not differentiable at 1.
(d) f is not continuous at 1.

QUESTION 4. What is the total number of local maxima and local minima of the function

$$f(x) = \begin{cases} (2+x)^3, & \text{if } x \in (-3,-1] \\ x^{2/3}, & \text{if } x \in (-1,2] \end{cases}$$

(a) 1
(b) 2
(c) 3
(d) 4

QUESTION 5. Let $f : \Re_{++} \to \Re$ is differentiable and $f(1) = 1$. Moreover, for every x

$$\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$$

Then $f(x)$ is
(a) $1/3x + 2x^2/3$
(b) $-1/5x + 4x^2/5$
(c) $-1/x + 2/x^2$
(d) $1/x$

QUESTION 6. An n-gon is a regular polygon with n equal sides. Find the number of diagonals (edges of an n-gon are not considered as diagonals) of a 10-gon.
(a) 20 diagonals
(b) 25 diagonals
(c) 35 diagonals
(d) 45 diagonals

QUESTION 7. The equation $x^7 = x + 1$
(a) has no real solution.
(b) has a real solution in the interval $(0, 2)$.
(c) has no positive real solution.
(d) has a real solution but not within $(0, 2)$.

QUESTION 8. $\lim_{n\to\infty} \left(\sqrt{n-1} - \sqrt{n}\right)$
(a) equals 1.
(b) equals 0.
(c) does not exist.
(d) depends on n.

QUESTION 9. A rectangle has its lower left hand corner at the origin and its upper right hand corner on the graph of $f(x) = x^2 + x^{-2}$. For which x is the area of the rectangle minimized?

(a) $x = 0$
(b) $x = \infty$
(c) $x = \left(\frac{1}{3}\right)^{1/4}$
(d) $x = 2^{1/3}$

QUESTION 10. Consider the system of equations

$$\alpha x + \beta y = 0$$
$$\mu x + \nu y = 0$$

α, β, μ and ν are i.i.d. random variables, each taking value 1 or 0 with equal probability. Consider the following propositions. (A) The probability that the system of equations has a unique solution is 3/8. (B) The probability that the system of equations has at least one solution is 1.

(a) Proposition A is correct but B is false.
(b) Proposition B is correct but A is false.
(c) Both Propositions are correct.
(d) Both Propositions are false.

Part II

Instructions.

• Answer **any four** of the following five questions in the space following the relevant question. No other paper will be provided for this purpose.

You may use the blank pages at the end of this booklet, marked **Rough work**, to do calculations, drawings, etc. Your "Rough work" will not be read or checked.

• Each question is worth 20 marks.

QUESTION 11. Suppose $\Re$ is given the Euclidean metric. We say that $f : \Re \to \Re$ is upper semicontinuous at $x \in \Re$ if, for every $\epsilon > 0$, there exists $\delta > 0$ such that $y \in \Re$ and $|x - y| < \delta$ implies $f(y) - f(x) < \epsilon$. We say that f is upper semicontinuous on $\Re$ if it is upper semicontinuous at every $x \in \Re$.

(A) Show that, f is upper semicontinuous on $\Re$ if and only if $\{x \in \Re \mid f(x) \geq r\}$ is a closed subset of $\Re$ for every $r \in \Re$.

(B) Consider a family of functions $\{f_i \mid i \in I\}$ such that $f_i : \Re \to \Re$ is upper semicontinuous on $\Re$ for every $i \in I$ and $\inf\{f_i(x) \mid i \in I\} \in \Re$ for every $x \in \Re$. Define $f : \Re \to \Re$ by $f(x) = \inf\{f_i(x) \mid i \in I\}$.

Show that $\{x \in \Re \mid f(x) \geq r\} = \cap_{i \in I}\{x \in \Re \mid f_i(x) \geq r\}$ for every $r \in \Re$.

(C) In the light of (A) and (B), state and prove a theorem relating the upper semicontinuity of f and the upper semicontinuity of all the functions in the family $\{f_i \mid i \in I\}$.

ANSWER.

QUESTION 12. Let $|.|$ be the Euclidean metric on $\Re$. Consider the function $f : \Re \to \Re$. Suppose there exists $\beta \in (0, 1)$ such that $|f(x) - f(y)| \leq \beta|x - y|$ for all $x, y \in \Re$. Let $x_0 \in \Re$. Define the sequence (x_n) inductively by the formula $x_n = f(x_{n-1})$ for $n \in \mathcal{N}$.

Show the following facts.

(A) (x_n) is a Cauchy sequence.

(B) (x_n) is convergent.

(C) The limit point of x, say x^*, is a fixed point of f, i.e., $x^* = f(x^*)$.

(D) There is no other fixed point of f.

ANSWER.

QUESTION 13. Let V be a vector space and $P : V \to V$ a linear mapping with range space $\mathcal{R}(P)$ and null space $\mathcal{N}(P)$.

P is called a projector if

(a) $V = \mathcal{R}(P) \oplus \mathcal{N}(P)$, and

(b) for every $u \in \mathcal{R}(P)$ and $w \in \mathcal{N}(P)$, we have $P(u + w) = u$.

In this case, we say that P projects V on $\mathcal{R}(P)$ along $\mathcal{N}(P)$.

Show the following facts.

(A) P is a projector if and only if it is idempotent.

(B) If U is a vector space and $X : U \to V$ is a linear mapping with $\mathcal{R}(P) = \mathcal{R}(X)$, then P is a projector if and only if $PX = X$.

(C) P is a projector if and only if $I - P$ is a projector.

Let W be a vector space and $A : V \to W$ a linear mapping. Let $B : W \to V$ be a linear mapping such that $ABA = A$.

Show the following facts.

(D) $\rho(A) = \rho(AB)$, where $\rho(.)$ denotes the rank of the relevant linear mapping.

(E) AB projects W on $\mathcal{R}(A)$.

ANSWER.

QUESTION 14. Given $x, y \in \Re^n$, define $(x, y) = \{tx + (1-t)y \mid t \in (0,1)\}$. We say that $C \subset \Re^n$ is a convex set if $x, y \in C$ implies $(x, y) \subset C$. We say that $f : \Re^n \to \Re$ is a concave function if $x, y \in \Re^n$ and $t \in (0,1)$ implies $f(tx + (1-t)y) \geq tf(x) + (1-t)f(y)$.

(A) Show that $f : \Re^n \to \Re$ is a concave function if and only if $H(f) = \{(x, r) \in \Re^n \times \Re \mid f(x) \geq r\}$ is a convex set in $\Re^n \times \Re$.

(B) Consider a family of functions $\{f_i \mid i \in I\}$ where $f_i : \Re^n \to \Re$ is a concave function for every $i \in I$. Suppose $\inf\{f_i(x) \mid i \in I\} \in \Re$ for every $x \in \Re^n$. Show that $f : \Re^n \to \Re$, defined by $f(x) = \inf\{f_i(x) \mid i \in I\}$ is a concave function.

(C) Consider concave functions $f_1 : \Re^n \to \Re$ and $f_2 : \Re^n \to \Re$. Define $f : \Re^n \to \Re$ by $f(x) = \max\{f_1(x), f_2(x)\}$. Is f necessarily a concave function? Provide a proof or counter-example.

(D) Show that, if $f : \Re^n \to \Re$ is a concave function, then $\{x \in \Re^n \mid f(x) \geq r\}$ is a convex set for every $r \in \Re$.

(E) Is the converse of (D) true? Provide a proof or counter-example.

ANSWER.

QUESTION 15. (A) An urn contains N balls, of which Np are white. Let S_n be the number of white balls in a sample of n balls drawn from the urn *without replacement*. Calculate the mean and variance of S_n.

(B) Let X and Y be jointly continuous random variables with the probability density function

$$f(x, y) = \frac{1}{2\pi} \exp\left[-\frac{1}{2}(x^2 + y^2)\right]$$

(a) Are X and Y independent?

(b) Are X and Y identically distributed?

(c) Are X and Y normally distributed?

(d) Calculate Prob $[X^2 + Y^2 \leq 4]$.

(e) Are X^2 and Y^2 independent random variables?

(f) Calculate Prob $[X^2 \leq 2]$.

(g) Find the individual density function of X^2.

ANSWER.

Rough Work

Rough Work

Rough Work