

## ENTRANCE EXAMINATION, 2016

M.Phil./Ph.D.

## INTERNATIONAL TRADE AND DEVELOPMENT

[ Field of Study Code : ITDP (106) ]

Time Allowed : 3 hours

Maximum Marks : 70

Question Nos. 1 and 2 are compulsory. In addition, attempt *any three* questions from the rest, i.e., from Question Nos. 3 to 7. All the parts of the chosen questions should be answered.

1. Derive the cumulative density function, the mean and the variance of the sampling distribution of the sample minimum of a random sample of size  $n$  from the exponential distribution

$$f(x) = \begin{cases} \theta e^{-\theta x}, & \text{if } x \geq 0 \text{ and } \theta > 0 \\ 0, & \text{otherwise} \end{cases}$$

Show that the sample minimum converges to 0 in probability.

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2. Find and classify all the critical points of the following function as relative minima, relative maxima or saddle points :

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$$f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$$

3. A consumer has utility function  $u(x, y) = xy$ , where  $x$  and  $y$  denote quantities of two goods  $X$  and  $Y$  respectively. Let  $p_x$  and  $p_y$  be the prices of goods  $X$  and  $Y$  respectively.

(a) Derive the Hicksian demands and the expenditure function  $E(p_x, p_y, u)$  of this consumer, where  $u$  is the level of utility/welfare of the consumer.

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(b) Assume the consumer is utility maximizing. Suppose, to start with, the prices of goods  $X$  and  $Y$  are  $p'_x$  and  $p'_y$  respectively, and the consumer's income is  $m' = E(p'_x, p'_y, u')$ , i.e., he has income that is exactly equal to the minimum amount required to achieve welfare level  $u'$  with prices  $p'_x$  and  $p'_y$ . Suppose a new policy regime is introduced whereby good  $X$  is subject to a tax  $t$  per unit

of the good purchased and good  $Y$  is subject to a subsidy  $s$  per unit of the good purchased, so that the new prices of goods  $X$  and  $Y$  are  $p'_x + t$  and  $p'_y - s$  respectively. There is no change in the consumer's income. Suitable diagrams can be employed to support answers to the questions below :

- (i) Employ the expenditure function to show under what condition the consumer is better off and under what condition the consumer is worse off under the new policy regime. 4
- (ii) Suppose the consumer is better off under the new regime. What is the maximum lump sum amount that he is willing to pay as a bribe to stop any effort to move back to the old regime? 4
- (iii) Suppose the new regime is implemented and the consumer is better off under it. Let  $u^*$  be the maximum utility the consumer derives in the new regime. There is now a proposition to remove the tax and subsidy scheme and switch back to the old regime. What is the minimum lump sum compensation that needs to be given to the consumer to ensure that he is no worse off after the switch from the new to the old regime? 4
4. (a) What is the Lucas surprise supply function? Derive it mathematically. What does it imply about the effectiveness of monetary policy? 10
- (b) Under what conditions can small menu costs create large real effects? Explain. 10
5. You have  $n$  iid observations  $(y_i, x_{1i}, x_{2i})$  and consider two alternative regression models
- (i)  $y_i = x'_{1i}\beta + e_{1i}$  and  $E(x_{1i} e_{1i}) = 0$
- (ii)  $y_i = x'_{2i}\beta + e_{2i}$  and  $E(x_{2i} e_{2i}) = 0$
- where  $x_{1i}$  and  $x_{2i}$  have at least some different regressors. You want to know whether model (i) or model (ii) fits the data better. Define variance of the two models— $\sigma_1^2 = E(e_{1i}^2)$  and  $\sigma_2^2 = E(e_{2i}^2)$ . You decide that the model with the smaller variance fits better. You decide to test for this by testing the hypothesis of equal fit  $H_0 : \sigma_1^2 = \sigma_2^2$  against the alternative of unequal fit  $H_1 : \sigma_1^2 \neq \sigma_2^2$ . For simplicity, suppose that errors are observable.
- (a) Construct an estimate  $\hat{\theta}$  of  $\theta = \sigma_1^2 - \sigma_2^2$ . 4
- (b) Find the asymptotic distribution of  $\sqrt{n}(\hat{\theta} - \theta)$  as  $n \rightarrow \infty$ . 4
- (c) Find an estimator of the asymptotic variance of  $\hat{\theta}$ . 4
- (d) Propose a test of asymptotic size  $\alpha$  of  $H_0$  against  $H_1$ . 4
- (e) Suppose the test accepts  $H_0$ . Write briefly what is your interpretation. 4

6. (a) Discuss the necessary and sufficient conditions under which the Metzler's paradox may exist. 10
- (b) State and demonstrate algebraically the conditions under which currency devaluation has a positive effect on trade balance. 10
7. (a) A family farm is managed entirely by John and his three brothers. The full production function is given below :

<i>Number of brothers working on the farm</i>	<i>Total output (in ₹)</i>
1	2,000
2	3,200
3	4,200
4	4,800

- (i) Each brother can decide to migrate to the city where a job in the formal sector pays ₹ 1,500 and a job in the urban informal sector pays ₹ 700. Assume that each brother is risk-neutral and seeks to maximize his own income. Assume the family is individualistic, agricultural output is shared equally and there are no remittances to or from any family member. How many brothers will migrate to the city if the probability of finding a formal job is  $\frac{5}{8}$ ? Assume that if a person is indifferent between migrating or not migrating, he will in fact migrate.
- (ii) Brothers who migrate, do not send any remittances back home but the family decides to send ₹ 300 to each brother who migrates anyway. How many brothers will migrate? Show the entire calculation and explain your answer. 10
- (b) Explain rigidity of wages in the modern sector in the context of the labour-turnover model. 10

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